Conditional Randomization

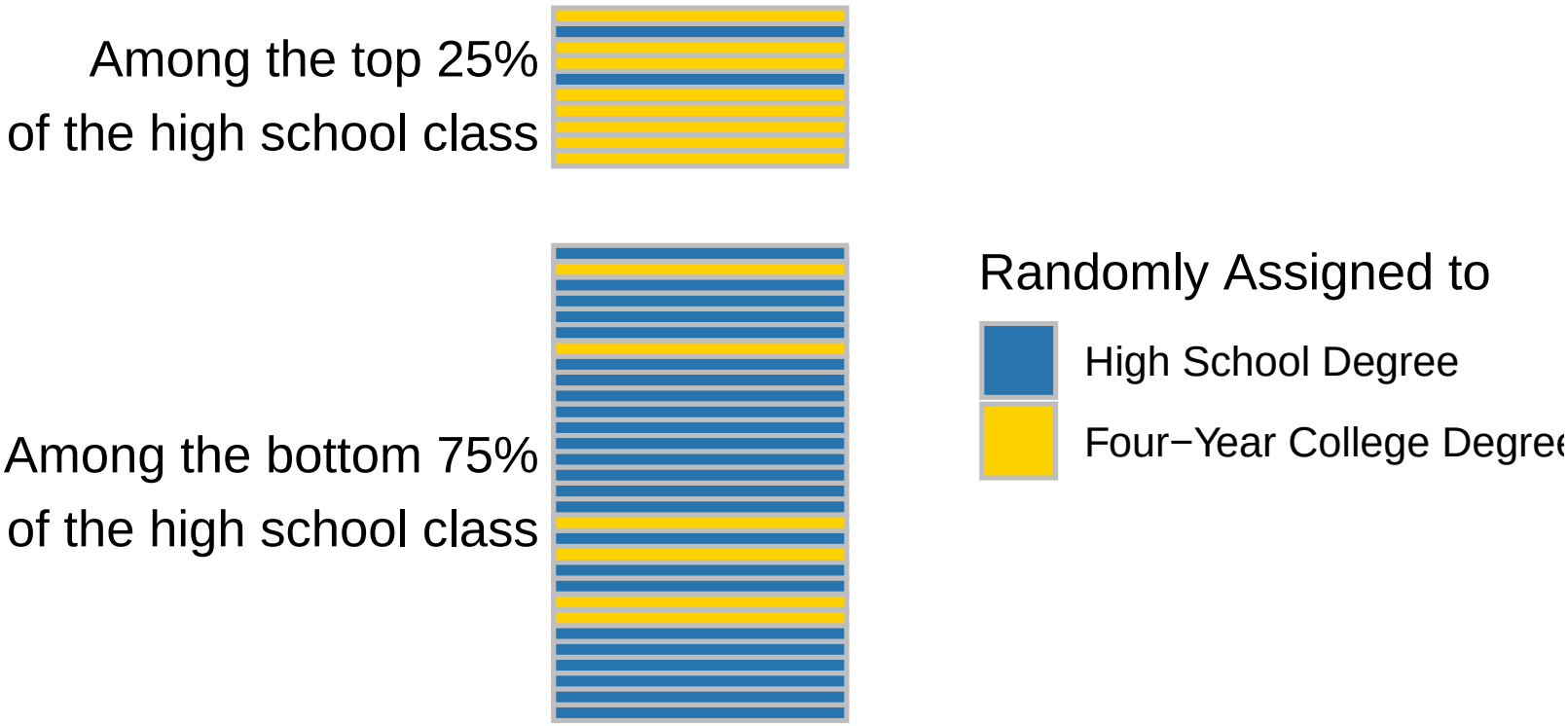
Ian Lundberg

Learning goals for today

At the end of class, you will be able to:

1. Define a conditionally randomized experiment
2. Define conditional exchangeability

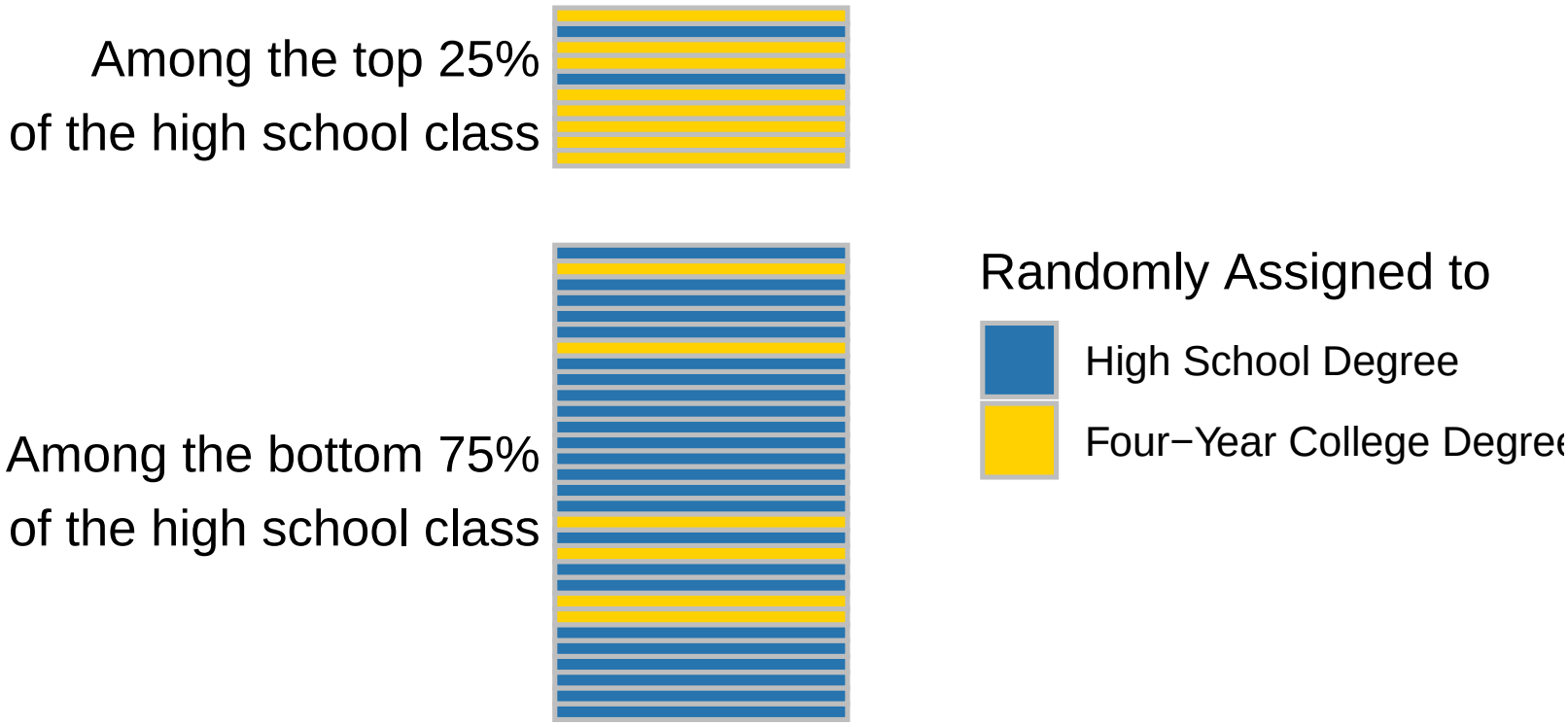
A hypothetical experiment: Conditional randomization



Outcome: Employed at age 40

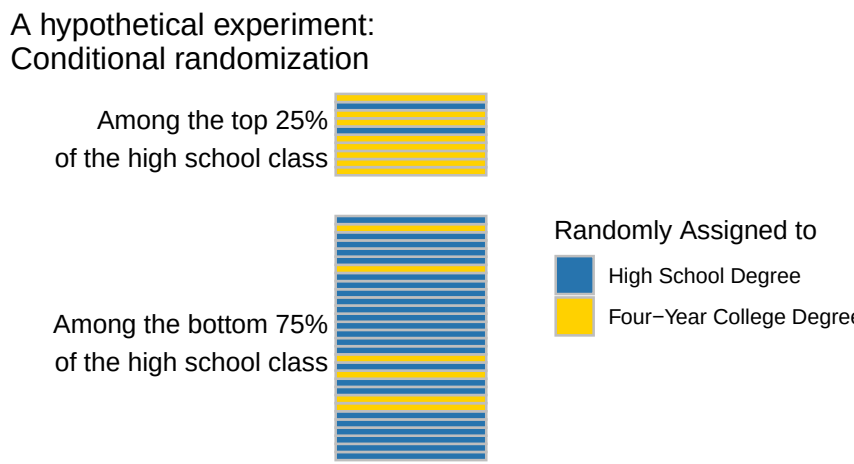
Does exchangeability hold? How would you analyze?

A hypothetical experiment: Conditional randomization



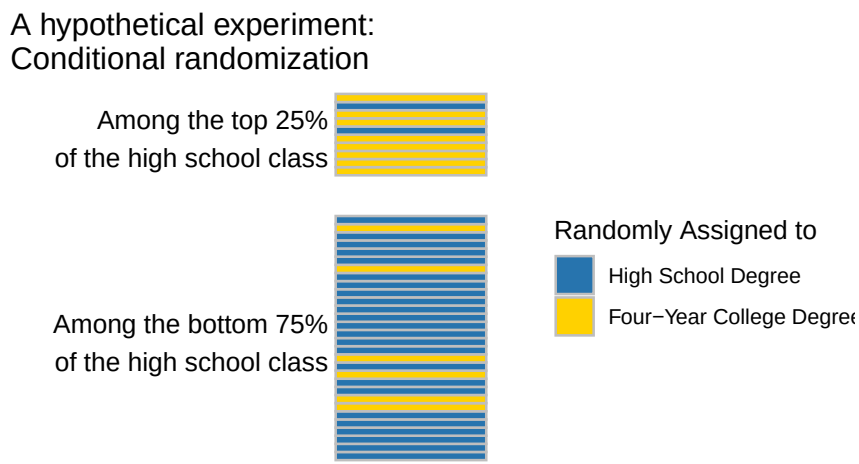
Outcome: Employed at age 40

Conditional randomization: Exchangeability does not hold



Conditional randomization: Exchangeability does not hold

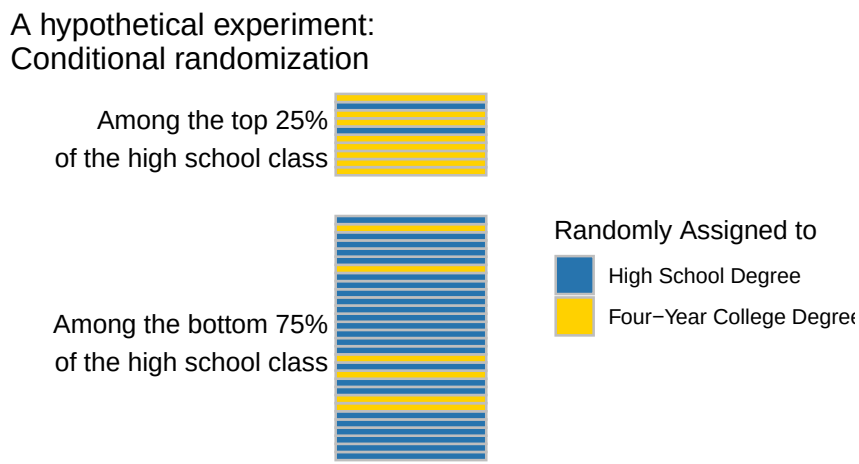
Treated units are more likely to have done well in high school



Conditional randomization: Exchangeability does not hold

Treated units are more likely to have done well in high school

Those who do well in high school are more likely to be employed at age 40 even without college

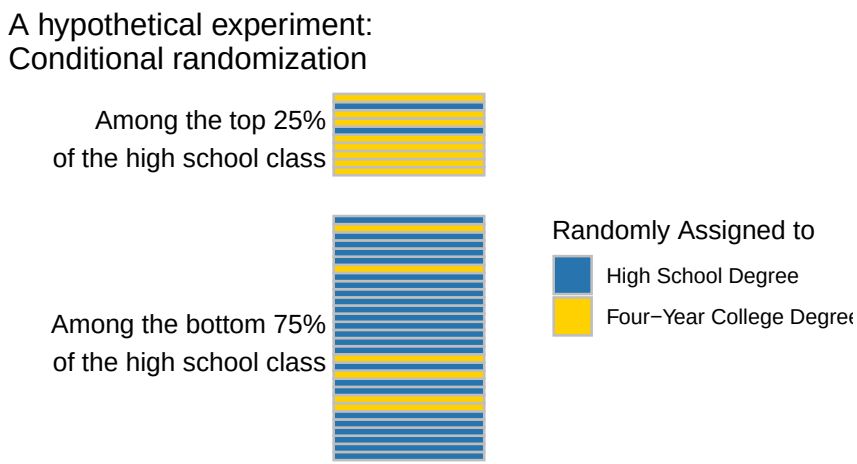


Conditional randomization: Exchangeability does not hold

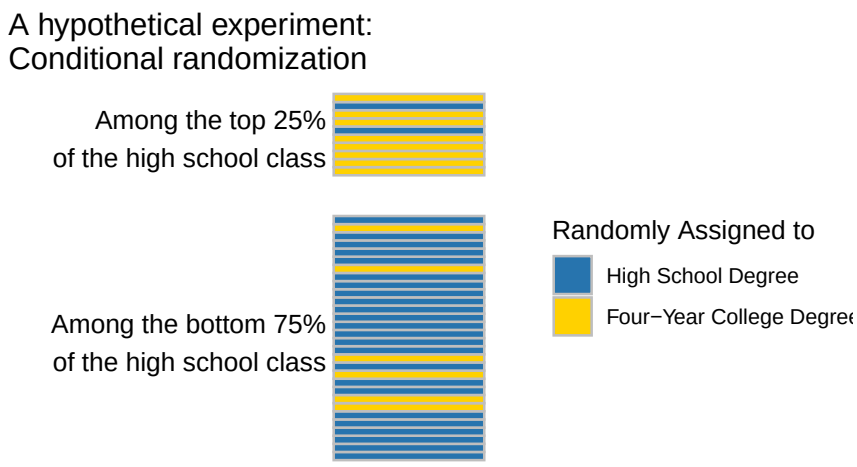
Treated units are more likely to have done well in high school

Those who do well in high school are more likely to be employed at age 40 even without college

$$\{Y^1, Y^0\} \not\perp\!\!\!\perp A$$

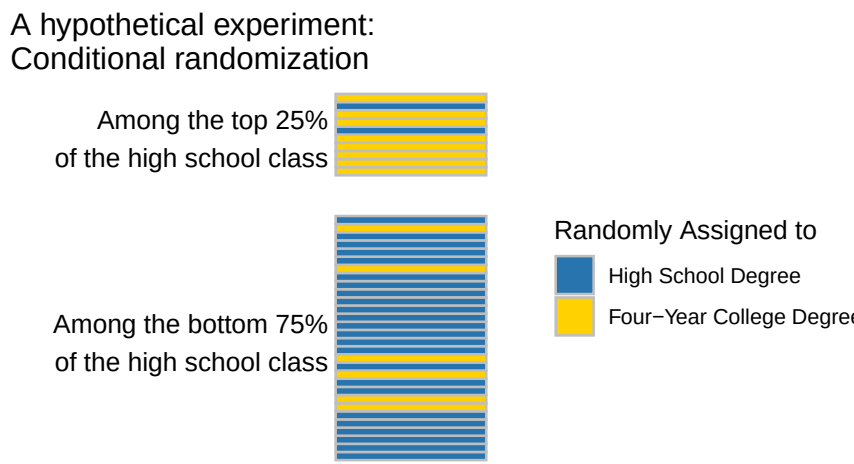


Conditional randomization: Analyze within subgroups



Conditional randomization: Analyze within subgroups

Among top 25%, simple random experiment.
Among bottom 75%, simple random experiment.

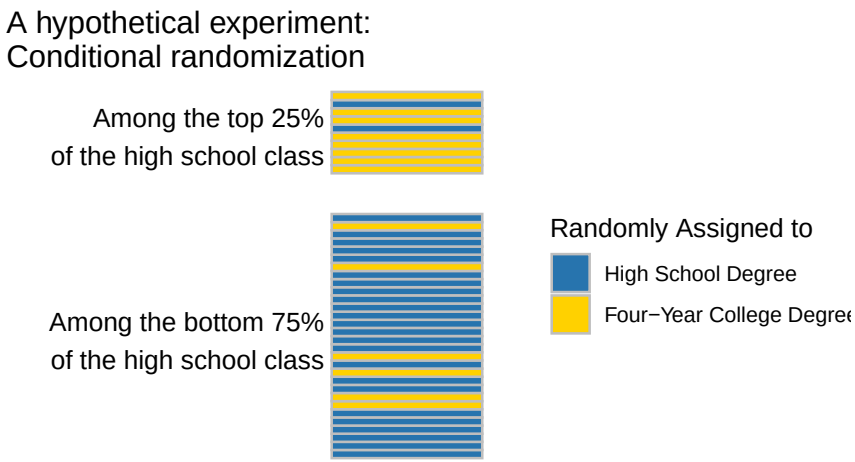


Conditional randomization: Analyze within subgroups

Among top 25%, simple random experiment.
Among bottom 75%, simple random experiment.

Conditional exchangeability:

$$\underbrace{\{Y^1, Y^0\}}_{\text{Potential Outcomes}} \underbrace{\perp\!\!\!\perp}_{\text{Are Independent of}} \underbrace{A}_{\text{Treatment}} \underbrace{\mid}_{\text{Within Subgroups}} \underbrace{X}_{\text{of X}}$$



Conditional average treatment effects

We get two estimates. Average effect of college on employment

- among those in the top 25% of their high school class
- among those in the bottom 75% of their high school class

These are **conditional average treatment effects**

$$\underbrace{\tau(x)}_{\substack{\text{Conditional}\\ \text{Average}\\ \text{Treatment}\\ \text{Effect}\\ \text{(CATE)}}} = \underbrace{\mathrm{E}}_{\substack{\text{Expected}\\ \text{value of}}} \left(\underbrace{Y^1 - Y^0}_{\text{treatment effect}} \quad \underbrace{|}_{\substack{\text{within the}\\ \text{subgroup}\\ \text{for whom}}} \quad \underbrace{\vec{X} = \vec{x}}_{\text{the predictors } \vec{X} \text{ take the value } \vec{x}} \right)$$

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