Directed Acyclic Graphs

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Why DAGs are worth knowing

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Why is that the case?

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2. Perhaps A and Y are related for other reasons

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DAGs formalize when (1) and not (2).

Learning goals for today

- fork structures
- collider structures
- causal reasoning and statistical independence

A hypothetical experiment in two population subgroups

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People who like exercise	People who don't like exercise

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People who like exercise	People who don't like exercise
Treatment 75% assigned an exercise coach for 1 month	**Treatment** 25% assigned an exercise coach for 1 month

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Outcome: How many pull-ups can they do?

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Outcome: How many pull-ups can they do?

Question for you:
Give 2 reasons why those assigned a coach can do more pull-ups

A hypothetical experiment in two population subgroups

$$
\begin{tikzcd}
\text{Assigned a Coach} \arrow[r] & \text{Pull-Ups} \\
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Nodes are random variables. **Edges** ($\rightarrow$) are causal relations

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The graph links causal assumptions to statistical dependence

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In this graph, (Assigned a Coach) and (Pull-Ups) are statistically dependent because of two open paths:

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In this graph, (Assigned a Coach) and (Pull-Ups) are statistically dependent because of two open paths:

- (Assigned a Coach) $\rightarrow$ (Pull-Ups)
  - a causal path: all arrows go one direction
- (Assigned a Coach) $\leftarrow$ (Likes Exercise) $\rightarrow$ (Pull-Ups)
  - a backdoor path containing a fork

A hypothetical experiment in two population subgroups

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How to study the causal effect (Assigned a Coach) $\rightarrow$ (Pull-Ups)?

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- pool the two estimates for an average causal effect estimate

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Terminology: We condition on [Likes Exercise]

Recap: Why DAGs are worth knowing

1. DAGs tell us when conditioning on $\vec{X}$ identifies a causal effect

$$\begin{tikzcd} A \arrow[r] & Y \\ \vec{X} \arrow[u] \arrow[ur] & \end{tikzcd}$$

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$$\begin{tikzcd} A \arrow[r] & Y \\ \vec{X} \arrow[u] \arrow[ur] & \end{tikzcd}$$

DAGs also tell us when conditioning can create problems.

Colliders: The sprinkler example

Example from Pearl, J. (1988). Probabilistic Reasoning in Intelligent Systems: Networks of Plausible Inference.

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- (Sprinklers) or (Rain) can make the grass wet

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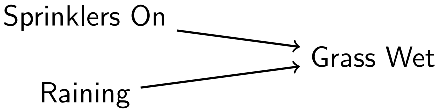

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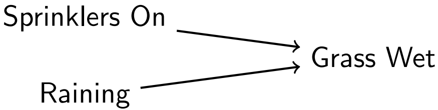

Questions:

- If (Sprinklers On = FALSE), does that help me predict (Raining)?
- If (Sprinklers On = FALSE) and (Grass Wet = TRUE), does that help me predict (Raining)?

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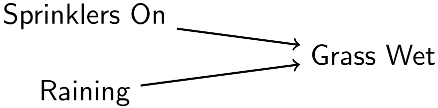

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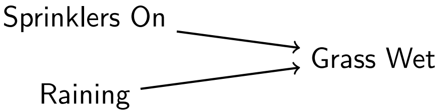

- (Grass Wet) is a **collider** (arrows collide $\rightarrow\leftarrow$)

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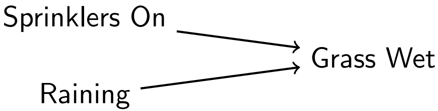

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  - marginal independence of (Sprinklers On) and (Raining)

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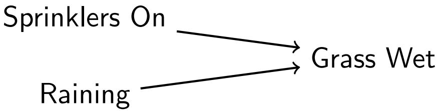

- (Grass Wet) is a **collider** (arrows collide $\rightarrow\leftarrow$)
- A collider blocks a path
  - marginal independence of (Sprinklers On) and (Raining)
- Conditioning on a collider opens the path
  - conditional dependence of (Sprinklers On) and (Raining) when restricting to times when (Grass Wet = `True`)

Using DAGs to identify causal effects: Game plan

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   - Create nodes for treatment and outcome

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   - Add other nodes

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   variables that jointly block all non-causal paths

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2. List all paths between treatment and outcome
3. Choose a sufficient adjustment set:
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   - a path is blocked if it contains an adjusted non-collider
   - a path is blocked if it contains an unadjusted collider
     (and no descendant of that collider is adjusted)
   - otherwise unblocked

Exercise 1

Find adjustment sets that identify the effect of A on Y

$$
\begin{tikzcd}
X_1 \arrow[r] \arrow[d] & A \arrow[r] & Y \\
X_2 \arrow[r] & X_3 \arrow[ur] &
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We can block the backdoor path in several ways:

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We can block the backdoor path in several ways:

- Condition on X_1: $A \leftarrow \boxed{X_1} \rightarrow X_2 \rightarrow X_3 \rightarrow Y$

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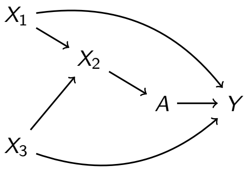
- Condition on X_3: $A \leftarrow X_1 \rightarrow X_2 \rightarrow \boxed{X_3} \rightarrow Y$
- Any combination of the above

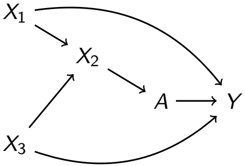
Exercise 2

Find 3 sufficient adjustment sets to identify $A \rightarrow Y$



Exercise 2

Find 3 sufficient adjustment sets to identify $A \to Y$



Answer: $\{X_2\}, \{X_1, X_3\}, \{X_1, X_2, X_3\}$

Exercise 3

What is the smallest adjustment set that identifies $A \to Y$?

$$\begin{tikzcd}
X_1 \arrow[dr] \arrow[ddrrr, bend left] & & & \\
& X_2 & & \\
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\end{tikzcd}$$

Answer: The empty set! Don't condition on anything.
The collider X_2 already blocks the path.

DAG in a realistic setting

To what extent does completing a 4-year college degree affect a person's future earnings?

Effect of a 4-year degree on future earnings

1) Draw a DAG

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degree earnings

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degree $\longrightarrow$ earnings

Effect of a 4-year degree on future earnings

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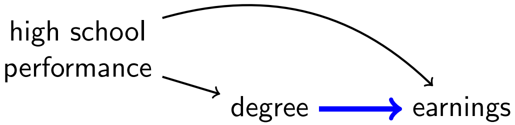

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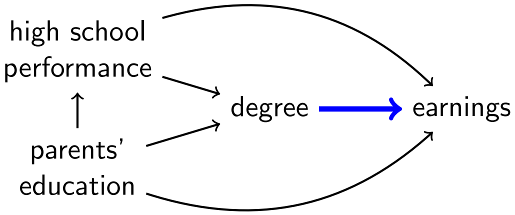
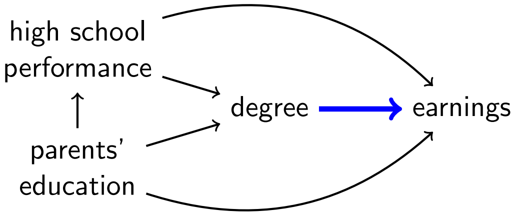


Effect of a 4-year degree on future earnings

2) List all paths between the treatment and outcome



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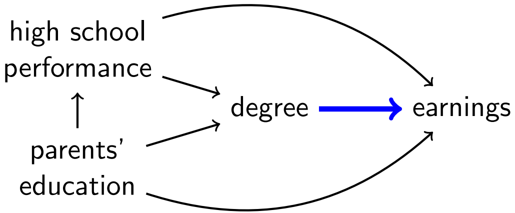

Causal paths

(degree) $\rightarrow$ (earnings)

Effect of a 4-year degree on future earnings

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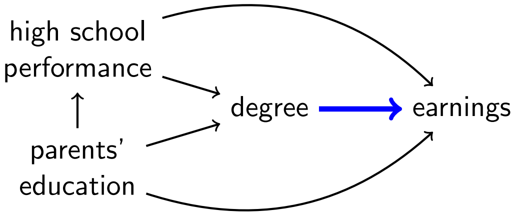


Causal paths

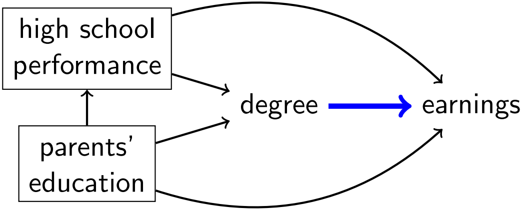
(degree) → (earnings)

Backdoor paths

(degree) ← (high school performance) → (earnings)
(degree) ← (parents' education) → (earnings)
(degree) ← (high school performance) ← (parents' education) → (earnings)

Effect of a 4-year degree on future earnings

3) Choose a sufficient adjustment set
{high school performance, parents' education}



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(degree) ← $\boxed{\text{high school performance}}$ ← $\boxed{\text{parents' education}}$ → (earnings)

DAGs: A promising path

- DAGs connect causal theories to statistical dependence
- Statistical dependence arises through causal paths
- Paths may contain two key structures
  - forks: $A \leftarrow B \rightarrow C$
    (A and C dependent if B unadjusted)
  - colliders: $A \rightarrow B \leftarrow C$
    (A and C dependent if B adjusted)
- Causal identification goal:
  choose a sufficient adjustment set so only the causal path of interest remains open
- Experimental analog:
  Among units who are identical on the sufficient adjustment set, we have a simple randomized experiment

DAGs: Words of warning

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Inference is only valid to the degree that the DAG holds

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- Your claim:
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It is important to reason about when the DAG may not hold

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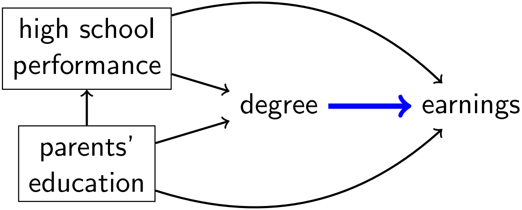

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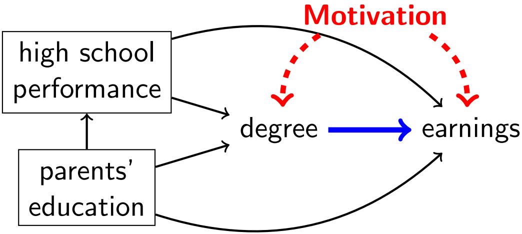

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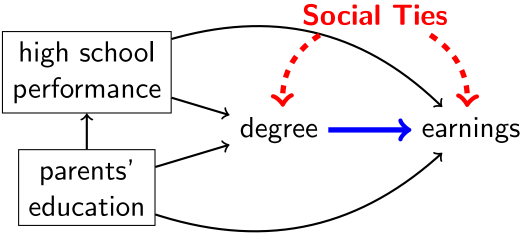

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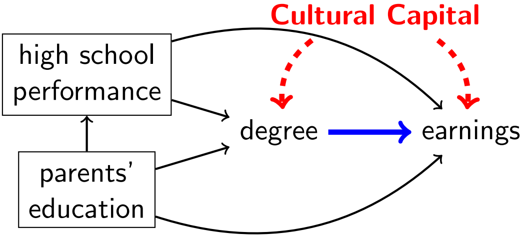

Resources to learn more

- Hernán, M.A., & J.M. Robins. 2020.
  Causal Inference: What If?
  Boca Raton: Chapman & Hall / CRC.
- Pearl, J., & Mackenzie, D. (2018).
  The Book of Why: The New Science of Cause and Effect.
  Basic Books.
- Pearl, J., Glymour, M., & Jewell, N. P. (2016).
  Causal Inference in Statistics: A Primer.
  John Wiley & Sons.
- Pearl, J. (2000).
  Causality.
  Cambridge University Press.