

Foundations of Causal Inference

(The Potential Outcomes Framework)

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Learning goals for today

- ▶ fundamental problem of causal inference
- ▶ potential outcomes
- ▶ recall mathematical concepts from probability
 - ▶ random variables
 - ▶ expectation
 - ▶ conditional expectation

Causal claims hinge on arguments, not on data



Left photo: By Fernando Frazão/Agência Brasil - http://agenciabrasil.ebc.com.br/sites/_agenciabrasil2013/files/fotos/1035034-_mg_0802_04.08.16.jpg, CC BY 3.0, <https://commons.wikimedia.org/w/index.php?curid=50548410>
Right photo: By Agencia Brasil Fotografias - EUA levam ouro na ginástica artística feminina; Brasil fica em 8 lugar, CC BY 2.0, <https://commons.wikimedia.org/w/index.php?curid=50584648>

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	Do you win gold if you:		Causal effect
	Swing	Do not swing	of swinging
Simone Biles	Yes (1)	?	?
Ian	?	No (0)	?

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Fundamental problem of causal inference

Holland 1986

Descriptive evidence



Person 1	lifespan	
Person 2		lifespan
Person 3	lifespan	
Person 4		lifespan
Person 5	lifespan	
Person 6	lifespan	
Person 7		lifespan
Person 8	lifespan	

Outcome
under
Mediterranean
diet

Outcome
under
standard
diet

Fundamental problem of causal inference

Holland 1986

Descriptive evidence



Causal claim



Person 1	lifespan	
Person 2		lifespan
Person 3	lifespan	
Person 4		lifespan
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lifespan	lifespan
lifespan	lifespan
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lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan

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Descriptive evidence



Causal claim



Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

Outcome
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Outcome
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lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan
lifespan	lifespan

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Descriptive evidence



Causal claim



Causal inference is a **missing data** problem

Person 1	lifespan	missing
Person 2	missing	lifespan
Person 3	lifespan	missing
Person 4	missing	lifespan
Person 5	lifespan	missing
Person 6	lifespan	missing
Person 7	missing	lifespan
Person 8	lifespan	missing

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Mathematical notation: Potential outcomes

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Y_i Outcome

Whether person i survived

Mathematical notation: Potential outcomes

Y_i Outcome

Whether person i survived

A_i Treatment

Whether person i ate a Mediterranean diet

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
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Y_i^a	Potential Outcome	Outcome person i would realize if assigned to treatment value a

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Examples:

$Y_{\text{Ian}} = \text{survived}$

Ian survived

Mathematical notation: Potential outcomes

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Examples:

$Y_{\text{Ian}} = \text{survived}$	Ian survived
$A_{\text{Ian}} = \text{MediterraneanDiet}$	Ian ate a Mediterranean diet

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
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Examples:

$Y_{\text{Ian}} = \text{survived}$	Ian survived
$A_{\text{Ian}} = \text{MediterraneanDiet}$	Ian ate a Mediterranean diet
$Y_{\text{Ian}}^{\text{MediterraneanDiet}} = \text{survived}$	Ian would survive on a Mediterranean diet

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
A_i	Treatment	Whether person i ate a Mediterranean diet
Y_i^a	Potential Outcome	Outcome person i would realize if assigned to treatment value a

Examples:

Y_{Ian}	= survived	Ian survived
A_{Ian}	= MediterraneanDiet	Ian ate a Mediterranean diet
$Y_{\text{Ian}}^{\text{MediterraneanDiet}}$	= survived	Ian would survive on a Mediterranean diet
$Y_{\text{Ian}}^{\text{StandardDiet}}$	= died	Ian would die on a standard diet

Mathematical notation: Potential outcomes

Y_i	Outcome	Whether person i survived
A_i	Treatment	Whether person i ate a Mediterranean diet
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Examples:

Y_{Ian}	= survived	Ian survived
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$Y_{\text{Ian}}^{\text{MediterraneanDiet}}$	= survived	Ian would survive on a Mediterranean diet
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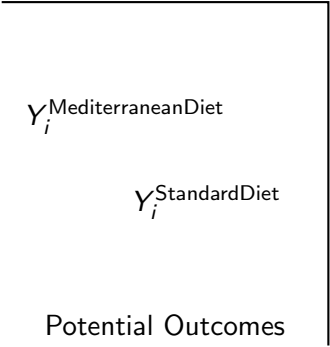
Discuss.

Which potential outcome is observed?

Which is counterfactual?

The consistency assumption

The consistency assumption

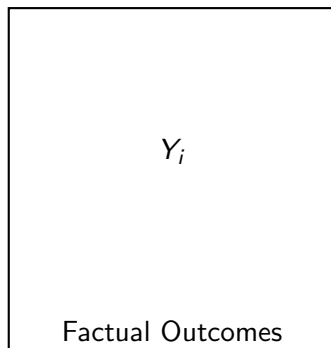
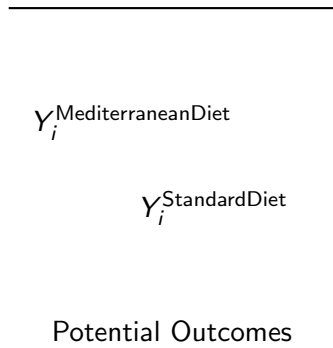


$Y_i^{\text{MediterraneanDiet}}$

$Y_i^{\text{StandardDiet}}$

Potential Outcomes

The consistency assumption



The consistency assumption

Consistency Assumption

$$Y_i^{A_i} = Y_i$$

$Y_i^{\text{MediterraneanDiet}}$

$Y_i^{\text{StandardDiet}}$

Potential Outcomes

Y_i

Factual Outcomes

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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The outcome for a random person is a **random variable**

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The outcome for a random person is a **random variable**

- ▶ Draw a random person from the population

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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The outcome for a random person is a **random variable**

- ▶ Draw a random person from the population
- ▶ Assign them a Mediterranean diet

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

$$Y_{\text{Ian}}^{\text{MediterraneanDiet}} = \text{survived}$$

The outcome for a random person is a **random variable**

- ▶ Draw a random person from the population
- ▶ Assign them a Mediterranean diet
- ▶ The outcome $Y^{\text{MediterraneanDiet}}$ is a random variable:
 - ▶ takes the value `survived` if we randomly sample some people
 - ▶ takes the value `died` if we randomly sample others

Mathematical notation: Potential outcomes are fixed

A person's potential outcome is a **fixed quantity**

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Check for understanding:

Does it make sense to write $V(Y_i^a)$? How about $V(Y^a)$

Notation: Expectation operator

The **expectation operator** $E()$ denotes the population mean

$$E(Y^a) = \frac{1}{n} \sum_{i=1}^n Y_i^a$$

The quantity Y^a inside the expectation must be a random variable

Notation: Expectation operator

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The quantity Y^a inside the expectation must be a random variable

A **conditional expectation** is denoted with a vertical bar

$$E(Y \mid A = a) = \frac{1}{n_a} \sum_{i:A_i=a} Y_i$$

Practice: How would you say this in English?

We might wonder how a person's earnings relate to whether they hold a college degree

1. $E(\text{Earnings} \mid \text{Degree} = \text{TRUE}) > E(\text{Earnings} \mid \text{Degree} = \text{FALSE})$

2. $E(\text{Earnings}^{\text{Degree}=\text{TRUE}}) > E(\text{Earnings}^{\text{Degree}=\text{FALSE}})$

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► Average earnings are higher among those with college degrees

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► On average, a degree causes higher earnings

Practice: How would you write this in math?

1. On average, students who do the homework learn more than those who don't
2. On average, doing the homework causes more learning

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Challenge exercise: Causal inference with interference

Person 1 races against Person 2.

Use potential outcomes math to write:

- ▶ If Person 1 gets a head start and Person 2 does not, then Person 1 wins.
- ▶ If neither gets a head start, then Person 2 wins.

Some things to consider:

- ▶ What is the unit of analysis?
- ▶ What is the outcome?
- ▶ What is / are the treatment(s)?

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Resources to learn more

- ▶ Hernán, M.A., & J.M. Robins. 2020.
[Causal Inference: What If?](#)
Boca Raton: Chapman & Hall / CRC.
- ▶ Imbens, G. W., & Rubin, D. B. 2015.
[Causal Inference in Statistics, Social, and Biomedical Sciences.](#)
Cambridge University Press.
- ▶ Brand, J. E. 2023.
[Overcoming the Odds: The Benefits of Completing College for Unlikely Graduates.](#)
Russell Sage Foundation.