Inverse Probability Weighting: Intuition in Two Periods

The causal problem and data

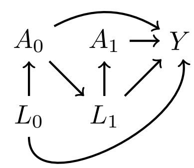
You teach a class of $n = 16$ elementary students. At each time point t, the student can either be below grade level ($L_t = 1$) or not below grade level ($L_t = 0$). You can assign extra support $A_t = 1$ (e.g., reading practice with a parent volunteer) to some but not all of the students; the untreated students have $A_t = 0$.

We will study this process over two time periods: $t = 0$ and $t = 1$. The outcome would be something defined after these periods, but for our purposes there will be no outcome. We will focus on how you would weight, which would apply to any outcome you might care about.

Here is how the causal process unfolds:

- At time 0, you see that 50% of the class is below grade level: $P(L_0 = 1) = 0.5$
- At every time period t, treatment is assigned so that
  - those below grade level $L_t = 1$ receive extra support $A_{t+1} = 1$ with probability 1
  - those not below grade level $L_t = 0$ receive extra support $A_{t+1} = 1$ with probability 0.5
- At every time period t, treatment affects the next confounder
  - if you receive support $A_t = 1$, then you will not fall behind $P(L_{t+1} = 1) = 0$
  - if you do not receive support $A_t = 0$, then you have a 50% chance of falling behind: $P(L_{t+1} = 1) = 0.5$

Try to draw a DAG for this process (an answer is on the next page).



The table below shows data from this causal process. Your task in Problems 1–3 is to fill in the weight columns with a pencil.

% latex table generated in R 4.5.2 by xtable 1.8-8 package % Mon Feb 23 08:42:48 2026

ID	L0	A0	L1	A1	w0	w1	w
1	1	1	0	1			
2	1	1	0	0			
3	1	1	0	1			
4	1	1	0	0			
5	1	1	0	1			
6	1	1	0	0			
7	1	1	0	1			
8	1	1	0	0			
9	0	1	0	1			
10	0	1	0	0			
11	0	1	0	1			
12	0	1	0	0			
13	0	0	1	1			
14	0	0	0	1			
15	0	0	1	1			
16	0	0	0	0			

Table 1: Weighting Two Time Periods: A Table to Fill In

1. Estimating $E(Y \mid do(A_0 = 1))$

Suppose we want to estimate the expected outcome under an intervention to set $A_0 = 1$. We plan to reweight the 12 units observed with $A_0 = 1$. What weight should we put on each of these 6 units?

Note that L_0 is a sufficient conditioning set.

2. Estimating $E(Y \mid do(A_1 = 1))$

Suppose we want to estimate the expected outcome under an intervention to set $A_1 = 1$. We plan to reweight the 9 units observed with $A_1 = 1$. What weight should we put on each of these 6 units?

Note that L_1 is a sufficient conditioning set.

3. Estimating $E(Y \mid do(A_0 = 1, A_1 = 1))$

Suppose we want to estimate the expected outcome under an intervention to set $A_0 = 1$ and $A_1 = 1$. We plan to reweight the 6 units observed in this condition. What weight should we put on each of these 6 units?

4. Check: Does your solution make sense?

Using your weight from 1.3,

- take a weighted average of L_0. Does your result match the factual distribution of L_0?
- take a weighted average of L_1. Does your result match the factual distribution of L_1?

Think about why you would get the answers above.