Non-existent outcomes in research on inequality: A causal approach

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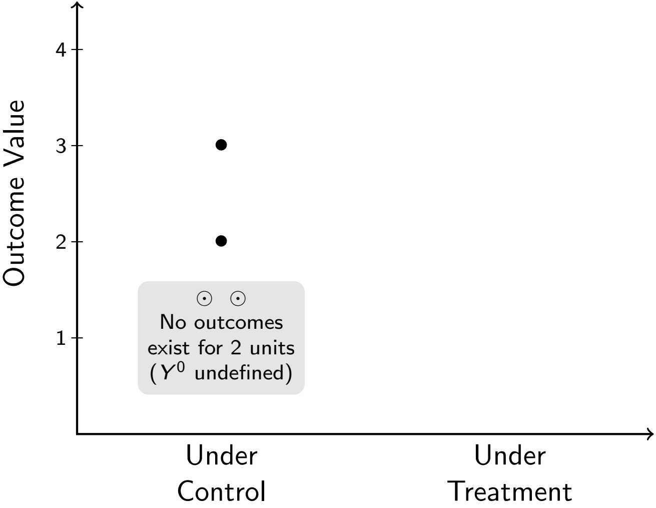
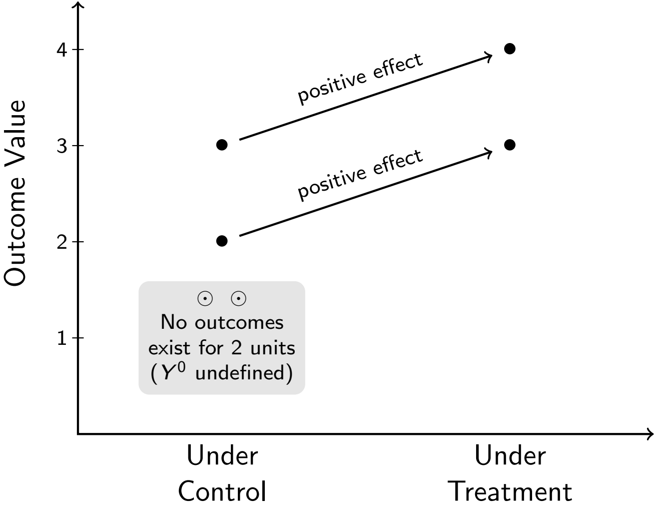
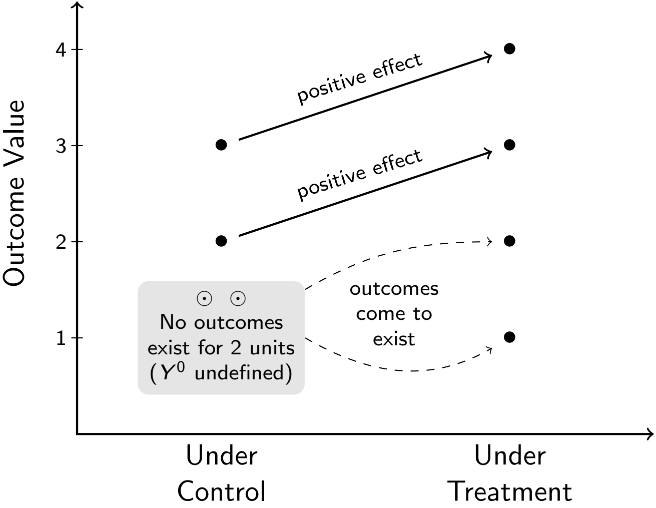
Some outcomes exist only for some people

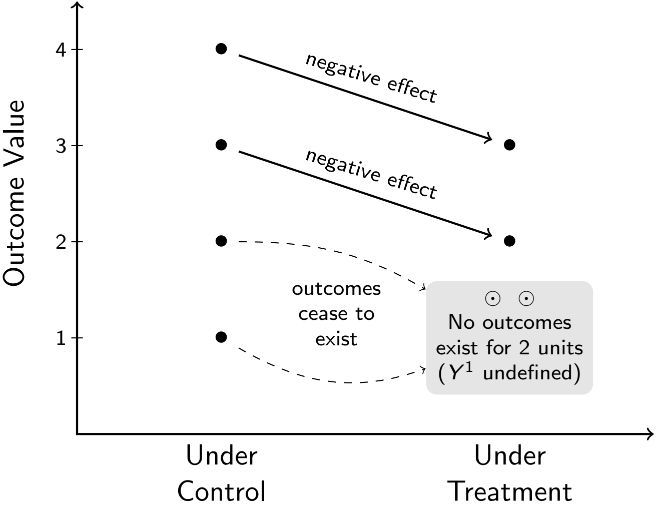
- only employed people have an hourly wage
- only married people can report marital satisfaction
- only people with children have descendants


Outcome Value
4
3
2
1
⊙ ⊙
No outcomes
exist for 2 units
(Y^0 undefined)
Under
Control
Under
Treatment


Outcome Value
4
3
2
1
positive effect
positive effect
⊙ ⊙
No outcomes exist for 2 units (Y^0 undefined)
Under Control
Under Treatment



Outcome Value
4
3
2
1
positive effect
positive effect
⊙ ⊙
No outcomes exist for 2 units (Y^0 undefined)
outcomes come to exist
Under Control
Under Treatment


Outcome Value
4
3
2
1
negative effect
negative effect
outcomes cease to exist
⊙ ⊙
No outcomes exist for 2 units (Y^1 undefined)
Under Control
Under Treatment

Our goals in this paper

When researchers drop missing outcomes,

- inequality can be obscured
- we may miss causal effects

We provide methods to study both:

- effects on outcome existence (employment)
- effects on outcome values (wage)

One concrete setting

Parenthood reduces hourly wages for women

(Budig & England 2001; Gough & Noonan 2013)

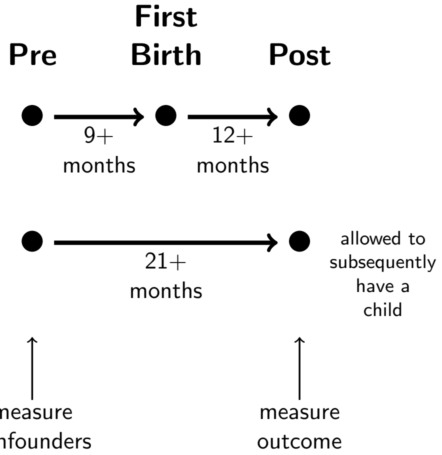
and increases wages for men

(Killewald 2013; Yu & Hara 2021)

The motherhood wage penalty may be disappearing over time

(Pal & Waldfogel 2016; Buchmann & McDaniel 2016; but see Jee et al. 2019)

Data: NLSY97



Parents

1,985 mothers
1,837 fathers

Non-Parents

	person-periods	persons
women	20,543	2,794
men	25,902	3,436

$$
\begin{aligned}
\text{log(Wage)} = \beta_0 &+ \beta_1(\text{Mother}) \\
&+ \beta_2(\text{Age}) \\
&+ \beta_3(\text{Married}) \\
&+ \beta_4(\text{Education}) \\
&+ \beta_5(\text{Work Experience}) \\
&+ \beta_6(\text{Full-Time}) \\
&+ \beta_7(\text{Tenure in Job}) \\
&+ \epsilon
\end{aligned}
$$

Maya

Maya

	if a mother	−	if not	=	effect
employment	_____	−	_____	=	_____
wage	_____	−	_____	=	_____

Maya

	if a mother	−	if not	=	effect
employment		−		=	
wage		−		=	

Maya

	if a mother	−	if not	=	effect
employment		−		=	
wage		−		=	

Maya

	if a mother	−	if not	=	effect
employment		−		=	0
wage		−		=	

Maya

	if a mother	−	if not	=	effect
employment		−		=	0
wage	\$30	−		=	

Maya

	if a mother	−	if not	=	effect
employment		−		=	0
wage	\$30	−	\$40	=	

Maya

	if a mother	−	if not	=	effect
employment		−		=	0
wage	\$30	−	\$40	=	−\$10

Mia

	if a mother	−	if not	=	effect
employment		−		=	
wage		−		=	

Mia

	if a mother	−	if not	=	effect
employment		−		=	
wage		−		=	

Mia

	if a mother	−	if not	=	effect
employment		−		=	
wage		−		=	

Mia

	if a mother		if not		effect
employment		$-$		$=$	-1
wage		$-$		$=$	

Mia

	if a mother		if not		effect
employment		−		=	-1
wage	??	−		=	

Mia

	if a mother	−	if not	=	effect
employment		−		=	-1
wage	??	−	\$20	=	

Mia

	if a mother	−	if not	=	effect
employment		−		=	-1
wage	??	−	\$20	=	??

Mia

	if a mother		if not		effect
employment		$-$		$=$	-1
wage	??	$-$	\$20	$=$	??

Principal Stratification
Frangakis & Rubin 2002; Zhang & Rubin 2003
For an intro, see Miratrix et al. 2018

Maya

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Mia

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Maya

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Nancy

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Mia

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Maya is a Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Nancy is a Non-Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Mia is a Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Maya is a Mother

if a mother	−	if not	=	effect
	−		=	0
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Nancy is a Non-Mother

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\$30	−	\$40	=	−\$10

Mia is a Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Maya is a Mother

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	−		=	0
\$30	−	\$40	=	−\$10

Mia is a Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

DROPPED

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Maya is a Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Nancy is a Non-Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Mia is a Mother

DROPPED

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Average Observed

\$30

Maya is a Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Nancy is a Non-Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Mia is a Mother

DROPPED

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Average Observed

\$30

Average Observed

\$30

Maya is a Mother

if a mother	$-$	if not	$=$	effect
	$-$		$=$	0
\$30	$-$	\$40	$=$	$-$\$10

Nancy is a Non-Mother

if a mother	$-$	if not	$=$	effect
	$-$		$=$	0
\$30	$-$	\$40	$=$	$-$\$10

Mia is a Mother

if a mother	$-$	if not	$=$	effect
	$-$		$=$	-1
??	$-$	\$20	$=$	??

Nia is a Non-Mother

if a mother	$-$	if not	$=$	effect
	$-$		$=$	-1
??	$-$	\$20	$=$	??

Maya is a Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

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if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	−\$10

Mia is a Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

1) Average effect of motherhood on employment

Maya is a Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	**−\$10**

Nancy is a Non-Mother

if a mother	−	if not	=	effect
	−		=	0
\$30	−	\$40	=	**−\$10**

Mia is a Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

Nia is a Non-Mother

if a mother	−	if not	=	effect
	−		=	−1
??	−	\$20	=	??

1) Average effect of motherhood on employment
2) Wage effect among those employed regardless

Goal 1: Effect on outcome existence

Causal Estimand

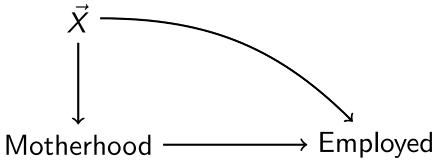
Define potential outcomes:
$S^1 =$ whether employed as a parent
$S^0 =$ whether employed as a non-parent

Causal estimand

$$\mathrm{E}(S^1 - S^0 \mid A = 1)$$

Goal 1: Effect on outcome existence

Causal Assumptions



Where $\vec{X}$ includes age, education, marital status, full-time employment, job tenure, work experience, and wage and employment each lagged by one year.

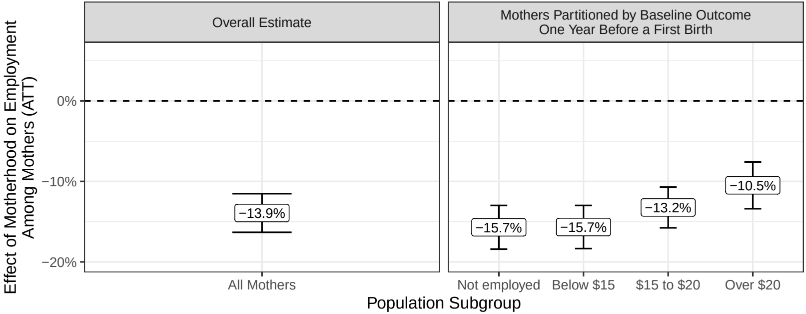
Goal 1: Effect on outcome existence

Estimation Strategy

- Regress Y on treatment and confounders
- Predict for all mothers average

$$\frac{1}{n_{\text{Mothers}}}\sum\left(\hat{Y}_i^1 - \hat{Y}_i^0\right)$$

Results for **mothers**



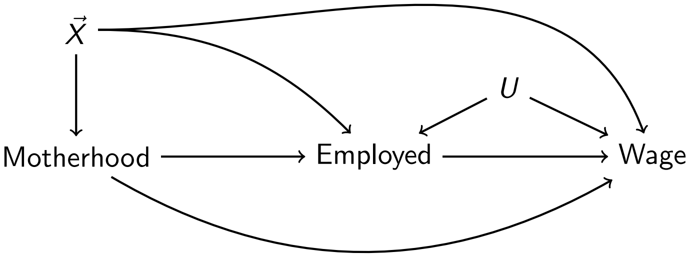
Goal 2: Effect on outcome value

Causal Estimand

$$\tau = \mathrm{E}\Bigg(\underbrace{Y^1 - Y^0}_{\substack{\text{Effect of Treatment} \\ \text{on Outcome Value}}} \quad \Big| \quad \underbrace{A = 1}_{\substack{\text{Among} \\ \text{the Treated}}} \; , \quad \underbrace{S^1 = S^0 = 1}_{\substack{\text{Whose Outcome} \\ \text{Exists Regardless} \\ \text{of Treatment}}} \Bigg) \tag{1}$$

Effect of motherhood on wage
among mothers who would be employed
as a parent or as a non-parent

Causal assumption: Exchangeability



Causal assumption: Monotonicity

$$S_i^1 \leq S_i^0 \quad \text{for all } i \qquad \text{(negative monotonicity)} \tag{2}$$

A woman who would be employed as a non-mother ($S_i^0 = 1$) would also be employed as a mother ($S_i^1 = 1$)

Causal assumption: Mean dominance

$$\begin{aligned}&\text{E}(Y^0 \mid \vec{X} = \vec{x}, S^0 = S^1 = 1) \geq \\ &\text{E}(Y^0 \mid \vec{X} = \vec{x}, S^0 = 1, S^1 = 0) \qquad \forall \vec{x}\end{aligned}$$

Women employed regardless of motherhood
would have higher mean wages as non-mothers
than women employed only if they were non-mothers

Identification: Making use of the assumptions

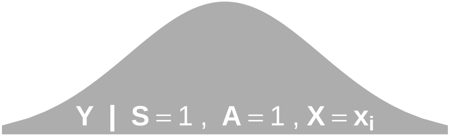
Recall that our goal is:

$$\begin{pmatrix}\text{mean factual}\\ \text{wage}\end{pmatrix} \quad - \quad \begin{pmatrix}\text{mean counterfactual wage}\\ \text{as non-mothers}\end{pmatrix}$$

among mothers employed regardless of motherhood.

Identification: Mean factual wage as mothers

We observe mothers with a wage distribution



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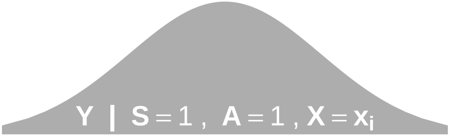
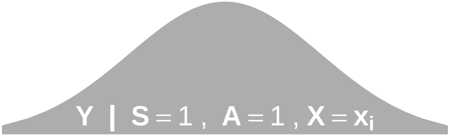


By assumption (monotonicity),
all would also be employed as non-mothers

Identification: Mean factual wage as mothers

We observe mothers with a wage distribution



By assumption (monotonicity),
all would also be employed as non-mothers

Estimate $\mathrm{E}(Y^1 \mid A = 1, S^0 = S^1 = 1, \vec{X})$ by the mean.

Identification: Counterfactual wage as non-mothers

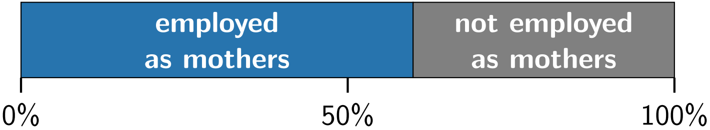
Identification: Counterfactual wage as non-mothers

We observe wages among non-mothers

Identification: Counterfactual wage as non-mothers

We observe wages
among non-mothers

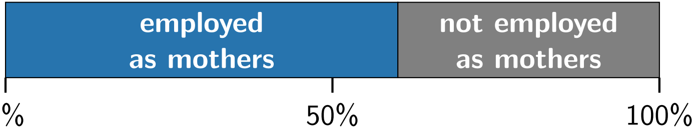
But some of them would not be employed as mothers



Identification: Counterfactual wage as non-mothers

We observe wages
among non-mothers

But some of them would not be employed as mothers



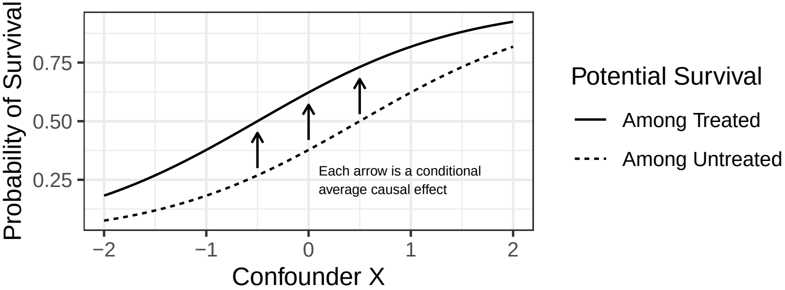
Two extreme possibilities:

Lower Bound:
Mean of Blue Region

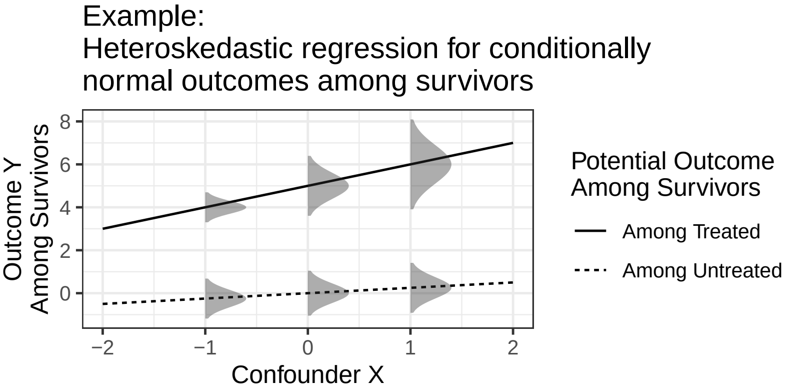
Upper Bound:
Mean of Blue Region

Estimation by regression and simulation

Example:
Logistic regression for survival



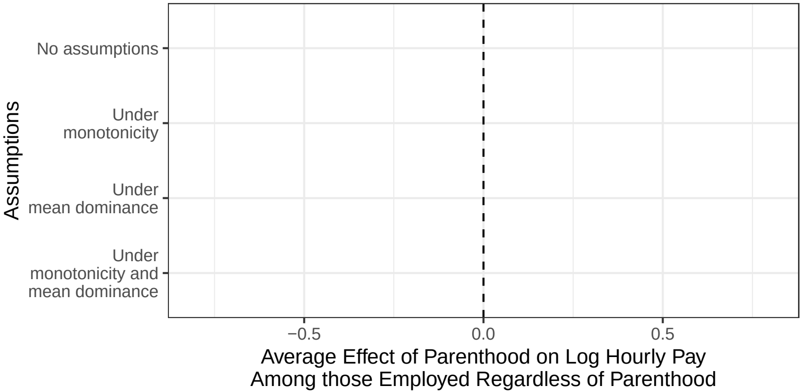
Estimation by regression and simulation



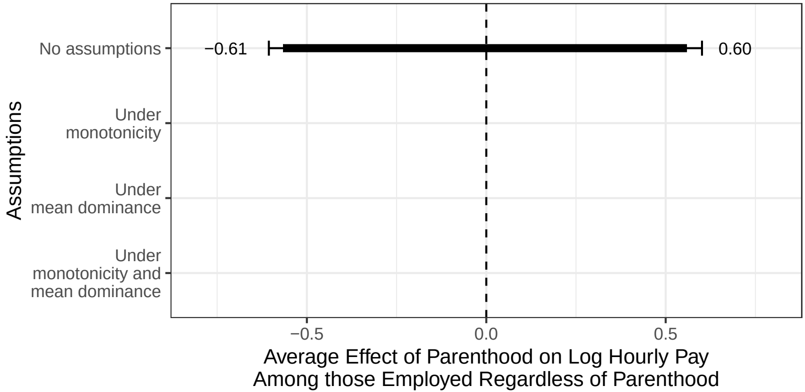
Review of our strategy

1. Define the causal estimands
2. Make causal assumptions (e.g., DAG + monotonicity)
3. Model Y among those with outcomes
4. Simulate a distribution for counterfactual estimation
5. Create bounds by considering extremes
   - Target subgroup are the **highest** paid
   - Target subgroup are the **lowest** paid

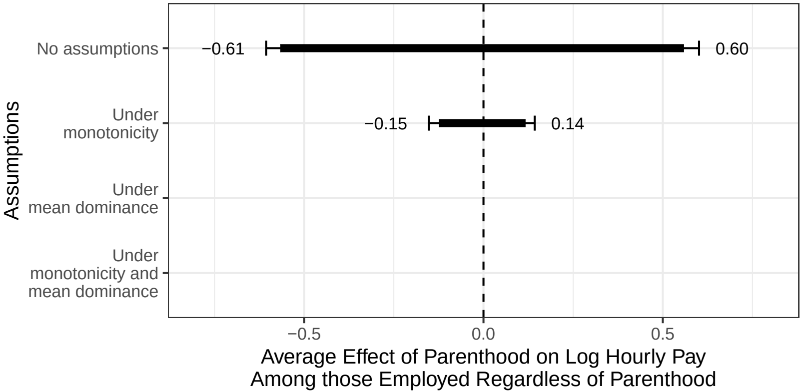
Results: Effect of motherhood on wages



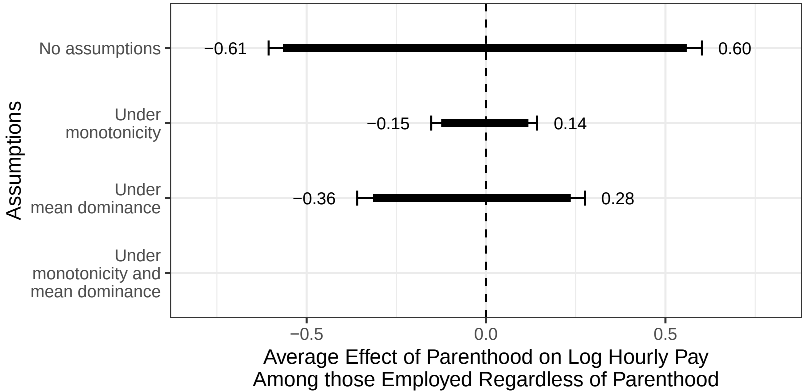
Results: Effect of motherhood on wages



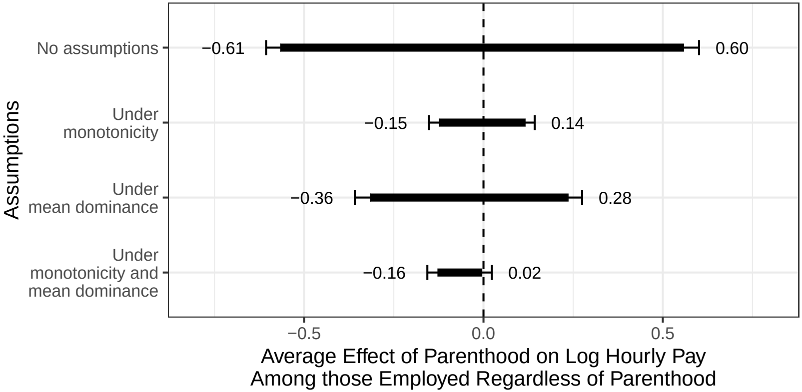
Results: Effect of motherhood on wages



Results: Effect of motherhood on wages



Results: Effect of motherhood on wages



Discussion: Building on past work

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When you can't point-identify,
make assumptions to set-identify (Manski 1995, 2003)

Discussion: Building on past work

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Principal stratification already did this (Frangakis & Rubin 2002)

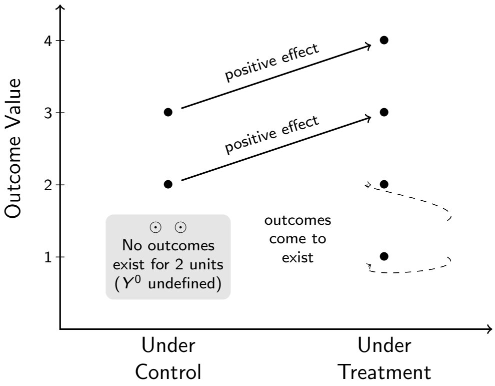
Discussion: Building on past work

When you can't point-identify,
make assumptions to set-identify (Manski 1995, 2003)

Principal stratification already did this (Frangakis & Rubin 2002)

Our new piece:
Regression + simulation for observational studies
with measured confounders

In social stratification,
many causes of outcome values
may also shape outcome existence



In social stratification,
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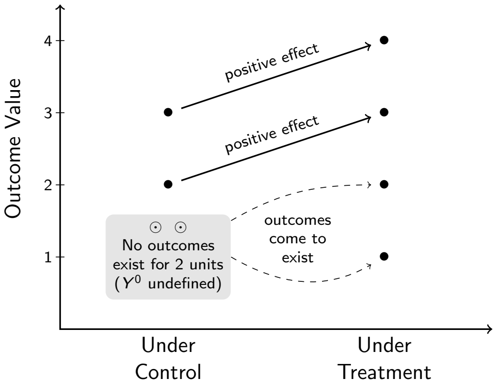

Thanks!

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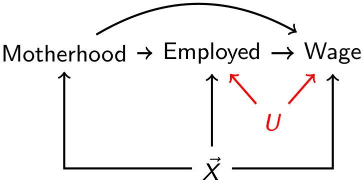
Draft of paper
tinyurl.com/nonexistentoutcomes

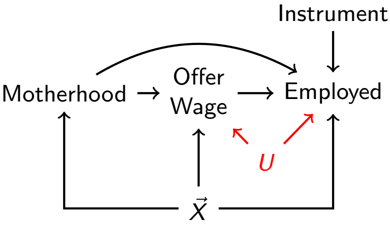
R package
ilundberg.github.io/pstratreg

Appendix Slides

Differences from Heckman selection model



Our DAG



Heckman DAG