Quantitative Data Analysis

SOCIOL 212B
Winter 2025

Lecture 4. Data-Driven Selection of an Estimator

Learning goals for today

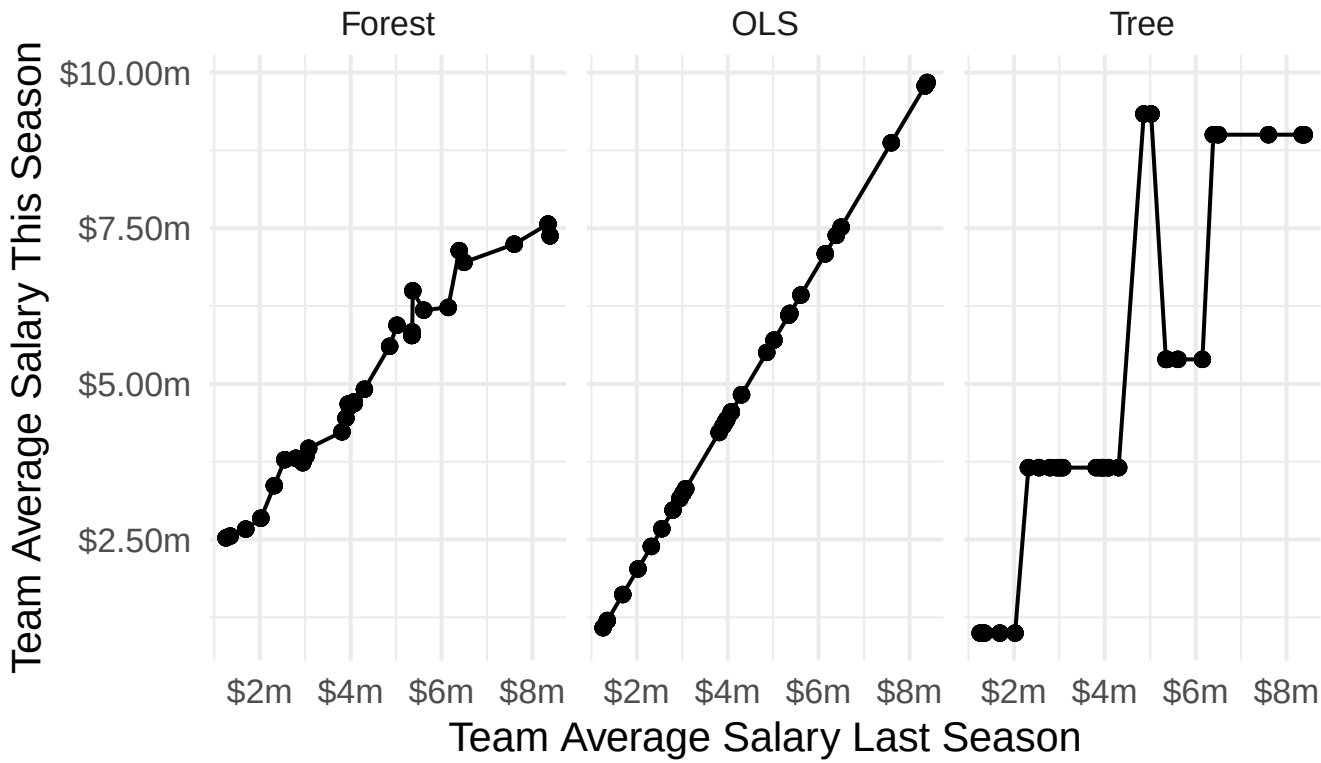
By the end of class, you will be able to

- Use individual prediction error as a metric to choose an algorithm that makes good group-level estimates
- Understand why we predict out of sample
- Carry out a sample split
- Explain the idea of cross-validation

On a different topic, we will close with a writing task.

Today's focus

We want to estimate $\mathrm{E}(Y \mid \vec{X})$. We have many algorithms:



How do we choose?

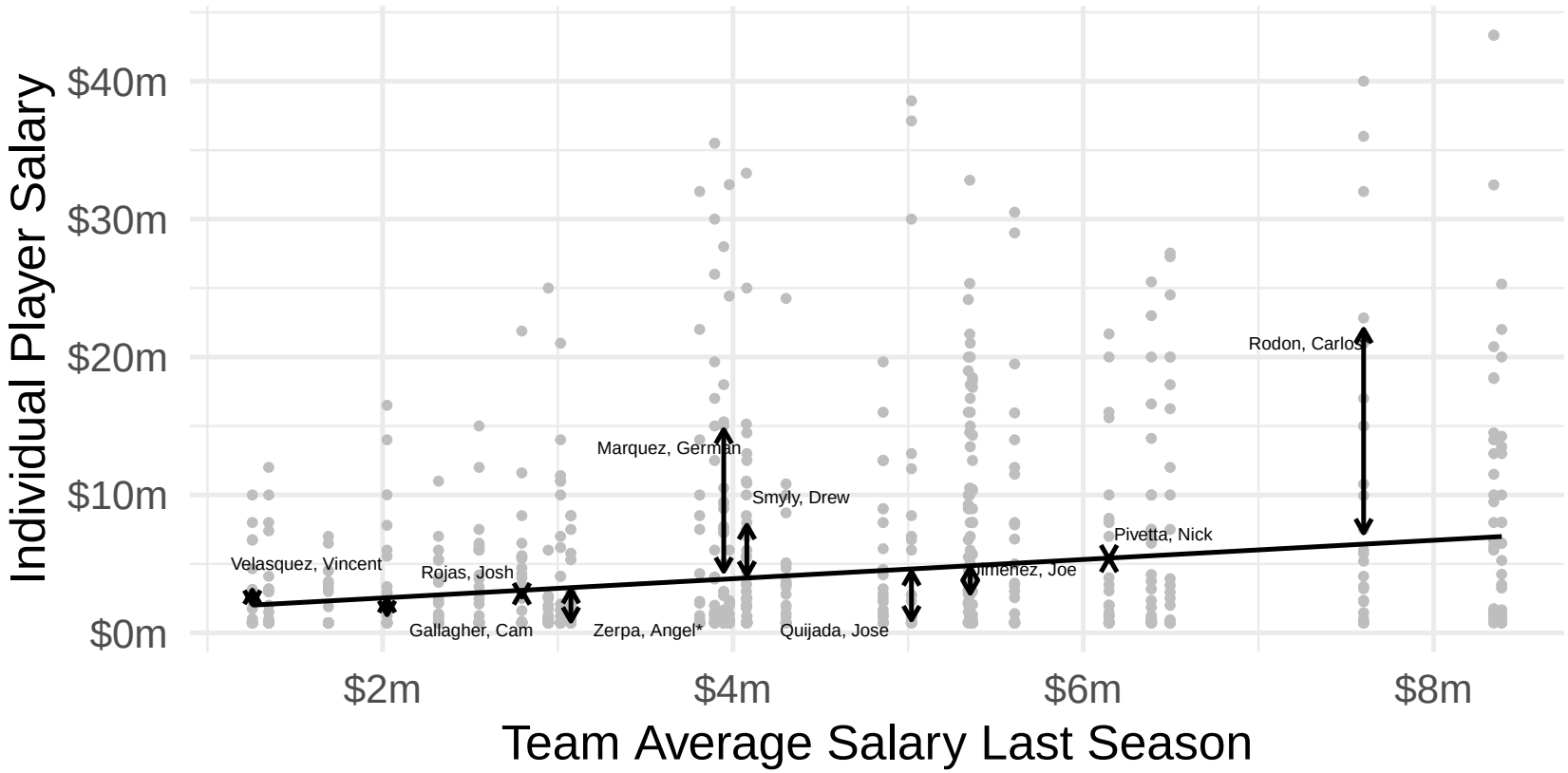
Concrete example: Evaluate model performance

Model player salaries this year as a linear function
of team average salaries last year

$$\text{E(Salary} \mid \text{Team)} = \alpha + \beta(\text{Team Average Salary Last Season})$$

Individual Prediction Errors

Randomly selected players highlighted for illustration



A score for individual-level predictive performance

$$R^2 = 1 - \frac{\overbrace{\mathrm{E}\left[\left(Y - \hat{f}(\vec{X})\right)^2\right]}^{\text{Expected Squared Prediction Error}}}{\underbrace{\mathrm{E}\left[(Y - \mathrm{E}(Y))^2\right]}_{\text{Variance of } Y}}$$

Limiting cases:

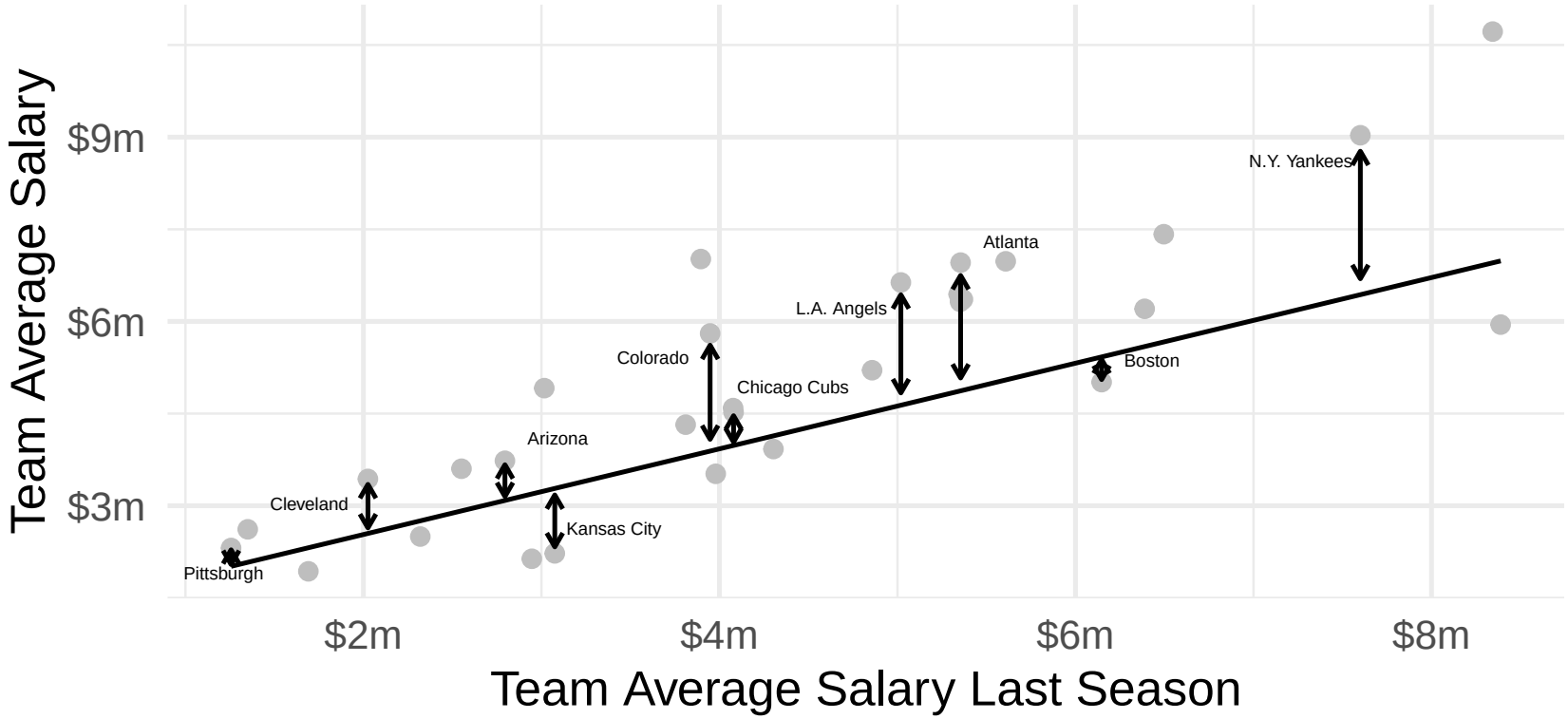
- 1 = perfect prediction
- 0 = predicted the overall mean for all cases

This example: $R^2 = 0.058$

What if the goal is to estimate for subgroups instead of to predict for individuals?

Group−Level Estimation Errors

Dots are true team mean salaries, excluding players from the learning sample in which the line was estimated



A score for group-level estimation performance

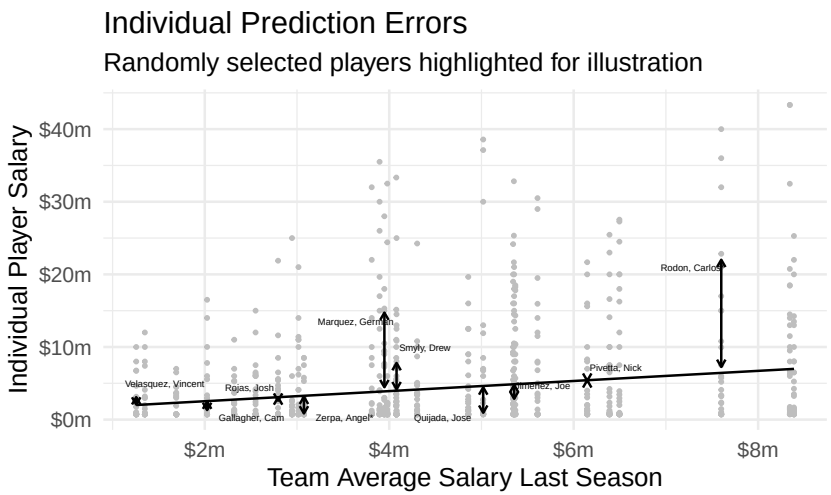
$$R^2_{\text{Group}} = 1 - \frac{\overbrace{\mathrm{E}\left[\left(\mathrm{E}(Y \mid \vec{X}) - \hat{f}(\vec{X})\right)^2\right]}^{\text{Expected Squared Estimation Error}}}{\underbrace{\mathrm{E}\left[\left(\mathrm{E}(Y \mid \vec{X}) - \mathrm{E}(Y)\right)^2\right]}_{\text{If Predicted E}(Y)\text{ for Everyone}}}$$

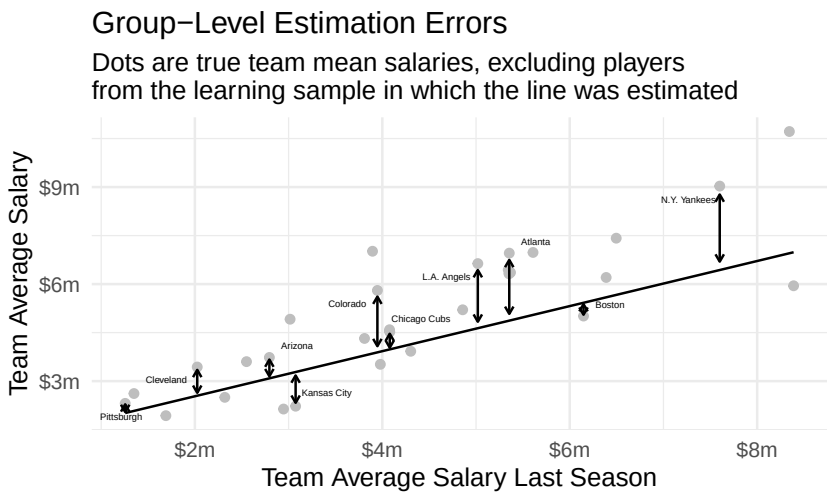
Variance of team-level prediction errors
over variance of team-level means[1]

In this example: $R^2_{\text{Group}} = 0.672$

[1] R^2_{Group} is not widely used, but I think it is useful here.

Individual prediction and subgroup estimation: Two very different goals?





$R^2 = 0.058$

$R^2_{\text{Group}} = 0.672$

Expected squared prediction error

Individual-level errors

$$\underbrace{\text{ESPE}(\hat{f})}_{\substack{\text{Expected Squared} \\ \text{Prediction Error}}} = \text{E}\left[\left(Y - \hat{f}(\vec{X})\right)^2\right]$$

In words:

The squared prediction error that occurs on average for a unit sampled at random from the population

Expected squared estimation error

Errors for estimating subgroup means

$$\underbrace{\text{ESEE}(\hat{f})}_{\substack{\text{Expected Squared} \\ \text{Estimation Error}}} = \text{E}\left[\left(\text{E}(Y \mid \vec{X}) - \hat{f}(\vec{X})\right)^2\right]$$

In words:

The squared error when estimating conditional means given $\vec{X}$, averaged over the distribution of the predictors $\vec{X}$

Within-group variance

Spread of individual outcomes within groups

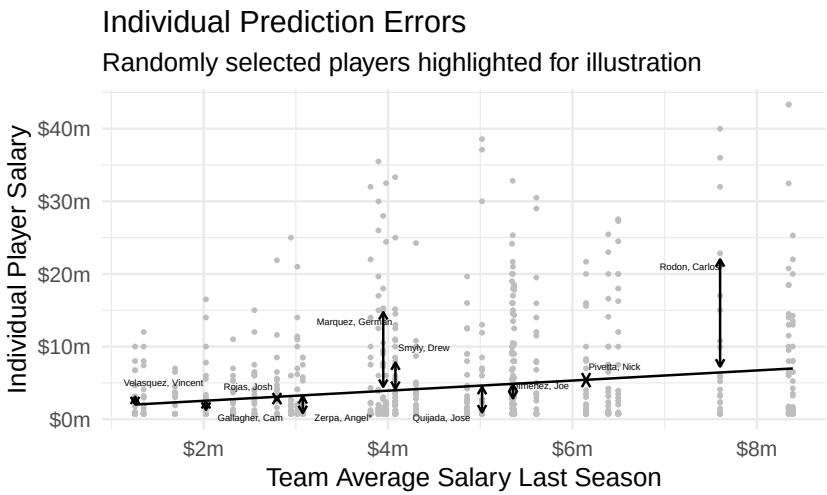
$$\mathsf{E}\left[\mathsf{V}\left(Y \mid \vec{X}\right)\right]$$

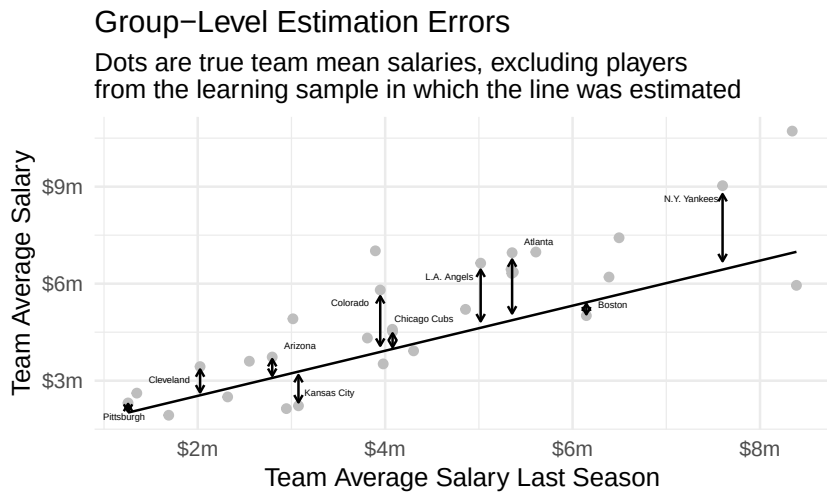
In words:

Within each subgroup defined by $\vec{X}$, outcomes vary.
This is the average value of the within-group variance,
with the average taken over $\vec{X}$

Decomposition of individual prediction error

$$\underbrace{\text{ESPE}(\hat{f})}_{\substack{\text{Expected Squared} \\ \text{Prediction Error}}} = \underbrace{\text{ESEE}(\hat{f})}_{\substack{\text{Expected Squared} \\ \text{Estimation Error}}} + \underbrace{\text{E}\left[\text{V}(Y \mid \vec{X})\right]}_{\substack{\text{Expected Within-} \\ \text{Group Variance}}}$$





Decomposition of individual prediction error

$$\underbrace{\text{ESPE}(\hat{f})}_{\substack{\text{Expected Squared} \\ \text{Prediction Error}}} = \underbrace{\text{ESEE}(\hat{f})}_{\substack{\text{Expected Squared} \\ \text{Estimation Error}}} + \underbrace{\text{E}\left[\text{V}(Y \mid \vec{X})\right]}_{\substack{\text{Expected Within-} \\ \text{Group Variance}}}$$

Suppose $\hat{f}_1$ and $\hat{f}_2$ are prediction functions
Suppose $\text{ESPE}(\hat{f}_1) < \text{ESPE}(\hat{f}_2)$
Then

$$\text{ESEE}(\hat{f}_1) \quad ? < \quad \text{ESEE}(\hat{f}_2)$$

Strategy:

- With many candidate algorithms $\hat{f}_1, \hat{f}_2, \ldots$
- choose the one that minimizes ESPE (individual prediction)
- It will also minimize ESEE (group estimation)

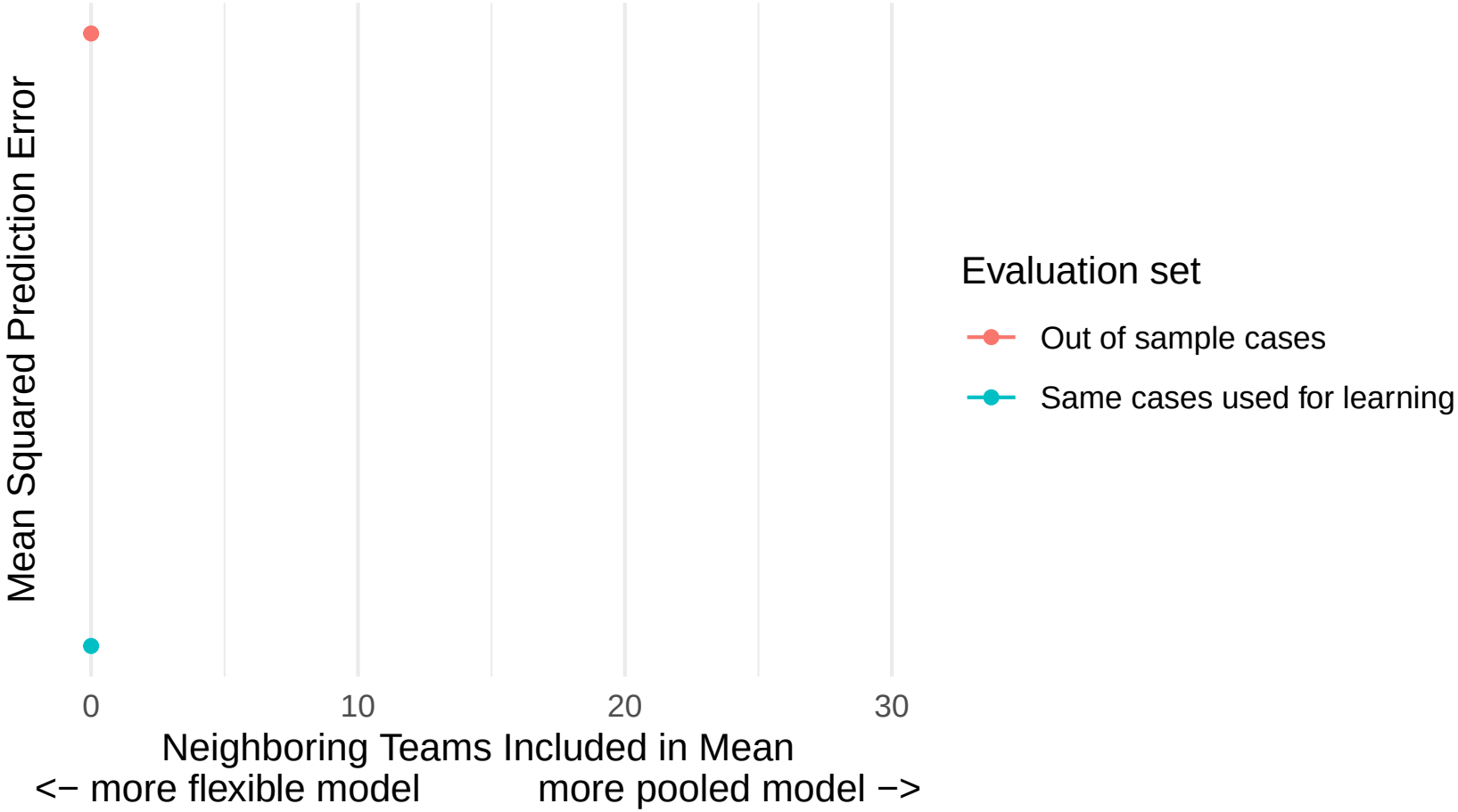
k-nearest neighbors

10 sampled players per team

- Dodger sample mean might be noisy
- Could pool with similar teams defined by past mean salary
  - Dodgers: 8.39m
  - 1st-nearest neighbor. NY Mets: 8.34m
  - 2nd-nearest neighbor. NY Yankees: 7.60m
  - 3rd-nearest neighbor. Philadelphia: 6.50m
- How does performance change with the number of neighbors included?
  - measured by mean squared prediction error

In−sample and out−of−sample measures of predictive performance

Nearest neighbor estimator applied to repeated samples of 10 players per team.
Curves are smoothed estimates over simulation results.



In−sample and out−of−sample measures of predictive performance

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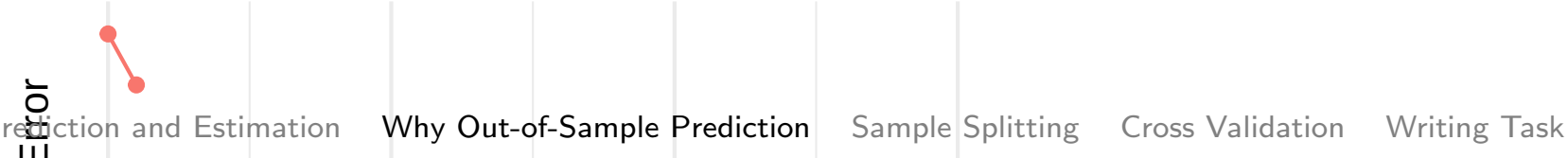
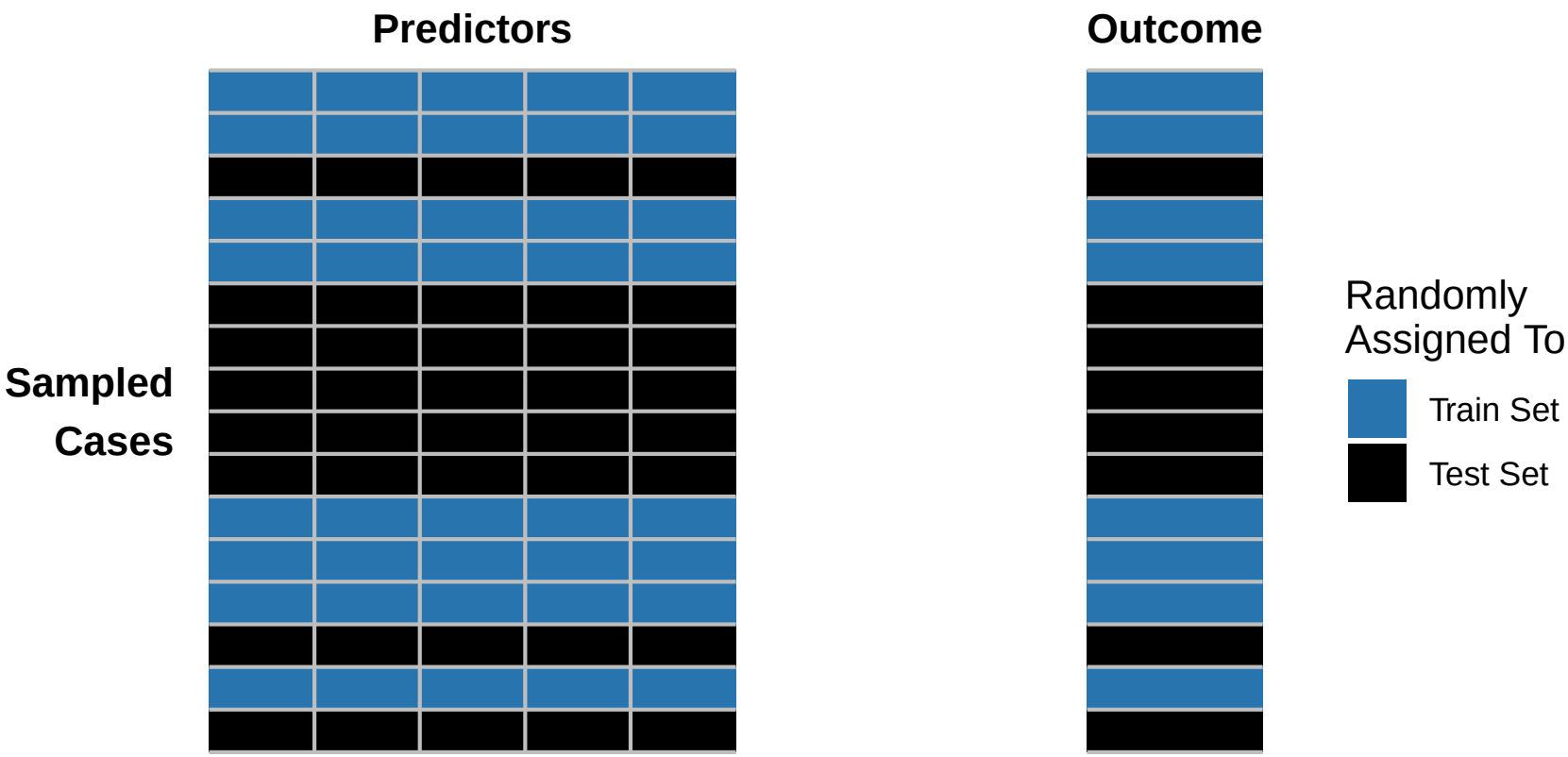
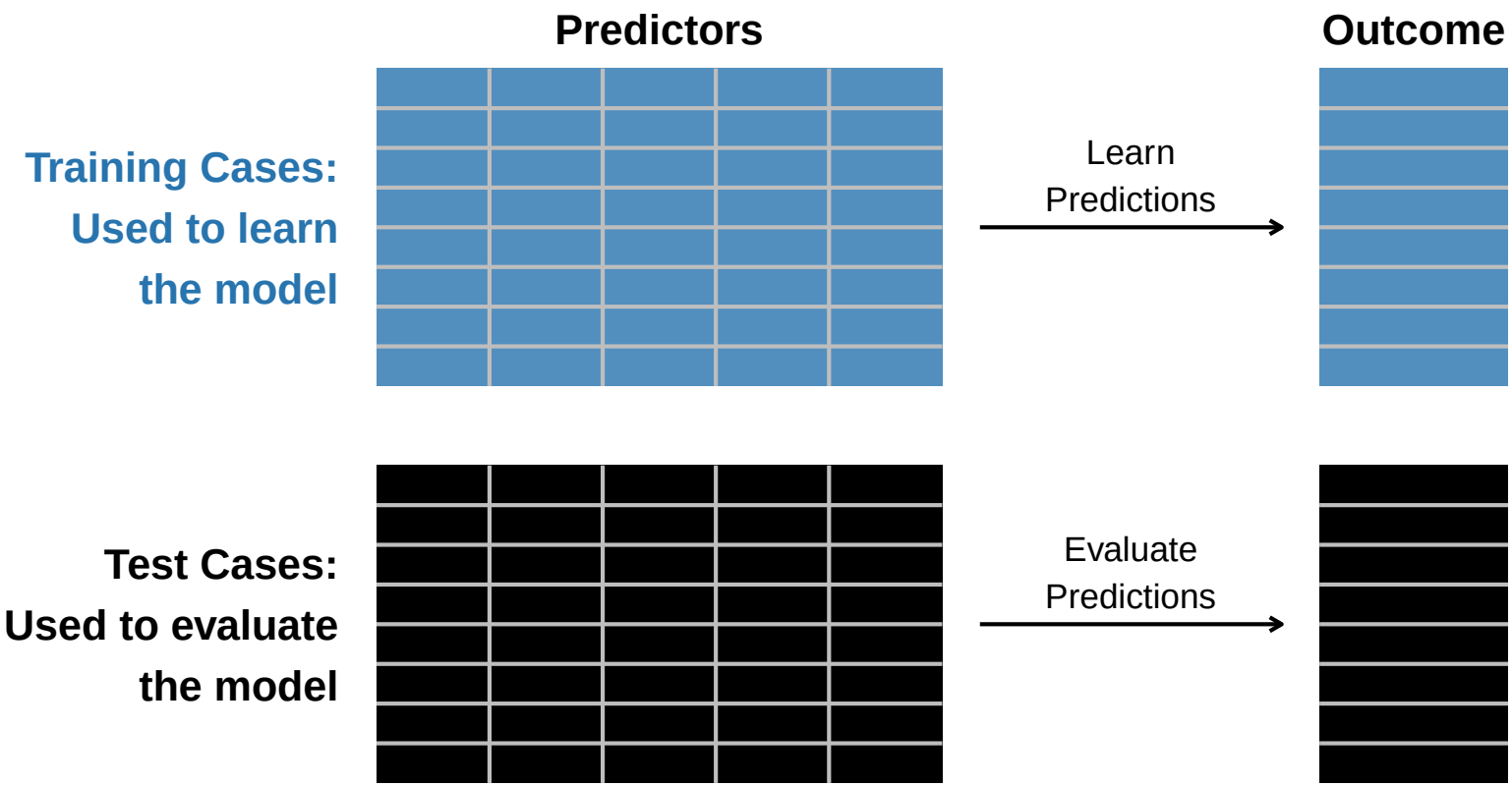


In−sample and out−of−sample measures of predictive performance

Nearest neighbor estimator applied to repeated samples of 10 players per team.
Curves are smoothed estimates over simulation results.

You have one sample.
How do you estimate out-of-sample performance?


Predictors
Outcome
Sampled Cases
Randomly Assigned To
Train Set
Test Set


Predictors
Outcome
Training Cases:
Used to learn
the model
Learn
Predictions
Test Cases:
Used to evaluate
the model
Evaluate
Predictions

Exercise: Conduct a sample split in code

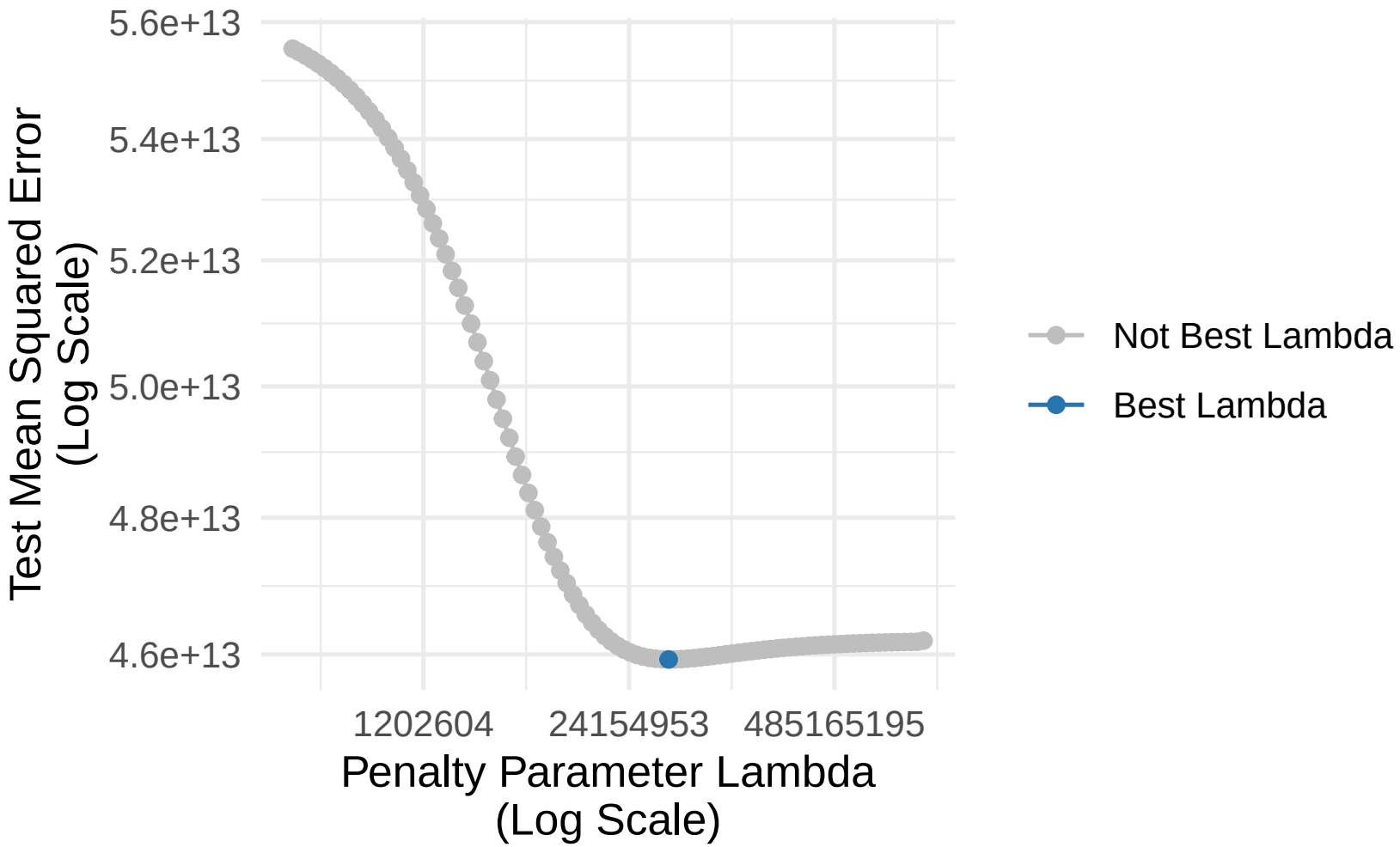
1. Sample 10 players per team
2. Take a 50-50 sample split stratified by team
3. Fit a linear regression in the train set
4. Predict in the test set
5. Report mean squared error

Sample splitting for parameter tuning: Ridge regression

$$\text{E}(\text{Player Salary} \mid \text{Team} = t) = \alpha + \beta_t$$

$$\left\{\hat{\alpha}, \hat{\vec{\beta}}\right\} = \underset{\tilde{\alpha}, \tilde{\vec{\beta}}}{\arg\min} \underbrace{\sum_{i=1}^{n} \left(Y_i - \left(\tilde{\alpha} + \tilde{\beta}_{t(i)}\right)\right)}_{\text{Prediction Error}} + \underbrace{\lambda \sum_{t} \tilde{\beta}_t^2}_{\text{Penalty}}$$

How do we choose λ?


5.6e+13
5.4e+13
5.2e+13
5.0e+13
4.8e+13
4.6e+13
Test Mean Squared Error
(Log Scale)
1202604
24154953
485165195
Penalty Parameter Lambda
(Log Scale)
Not Best Lambda
Best Lambda

Cross Validation

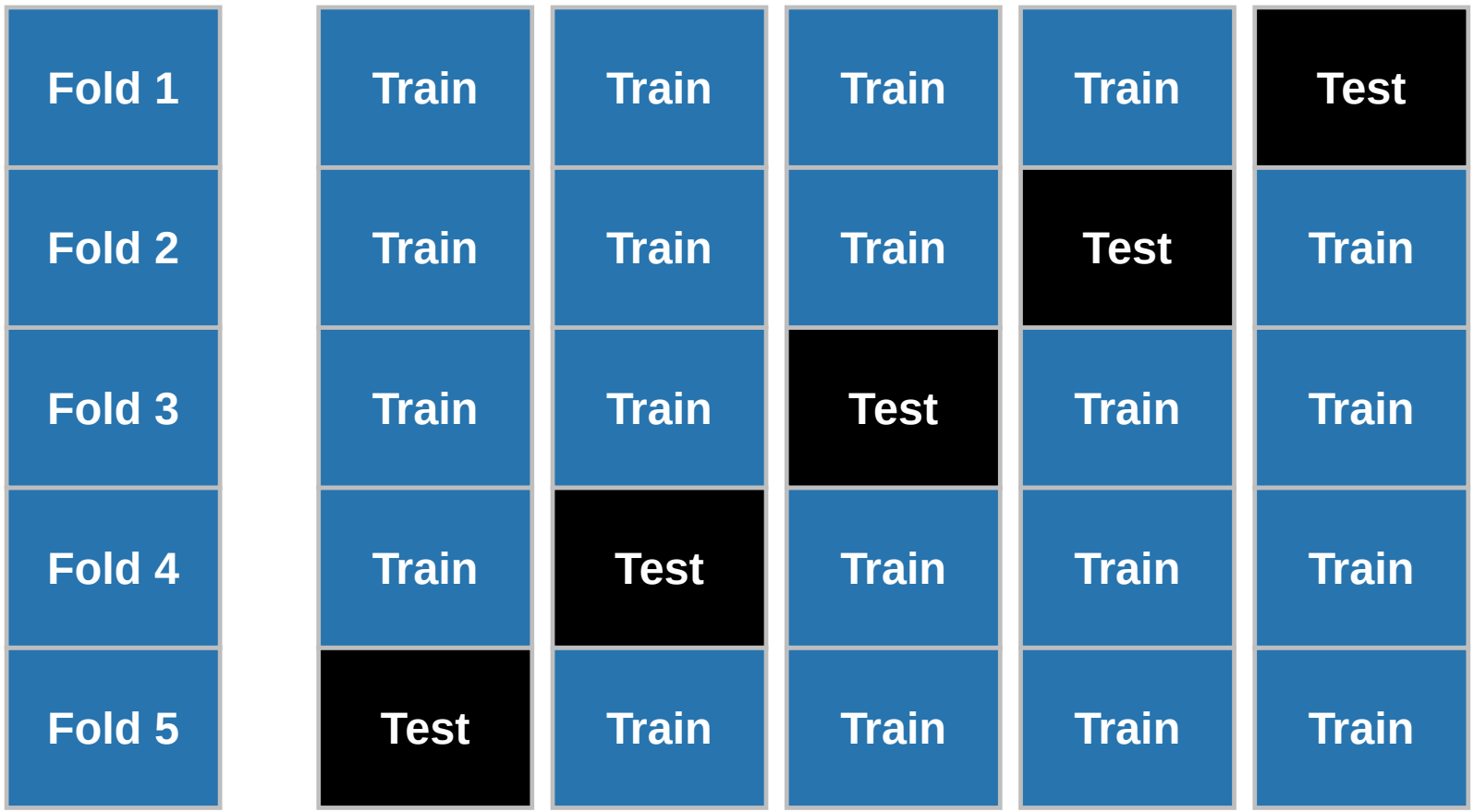
A train test split loses lots of data to testing.

Is there a way to bring it back?

Cross Validation

Randomize to 5 folds

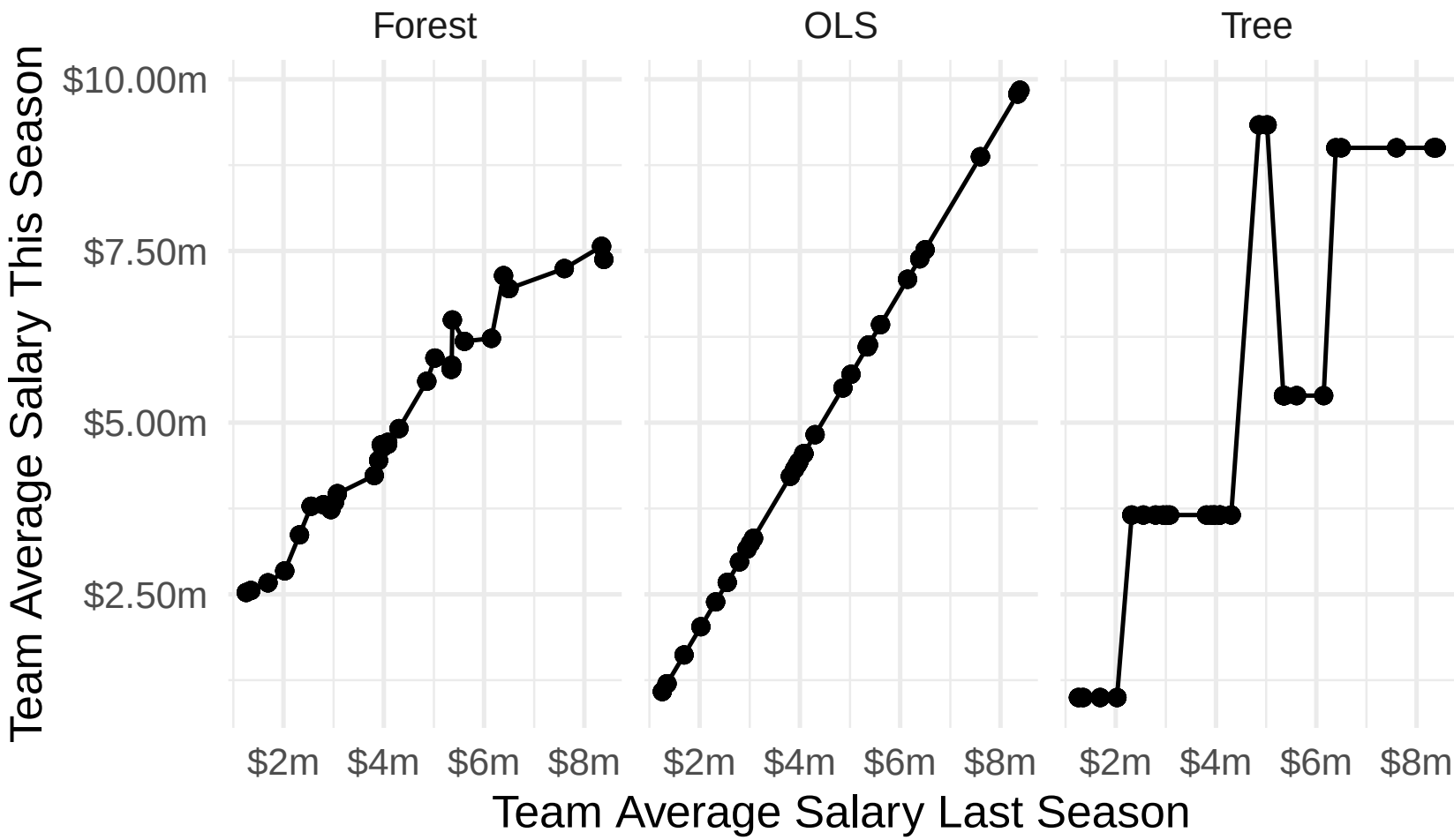
Iteratively use each as the test set

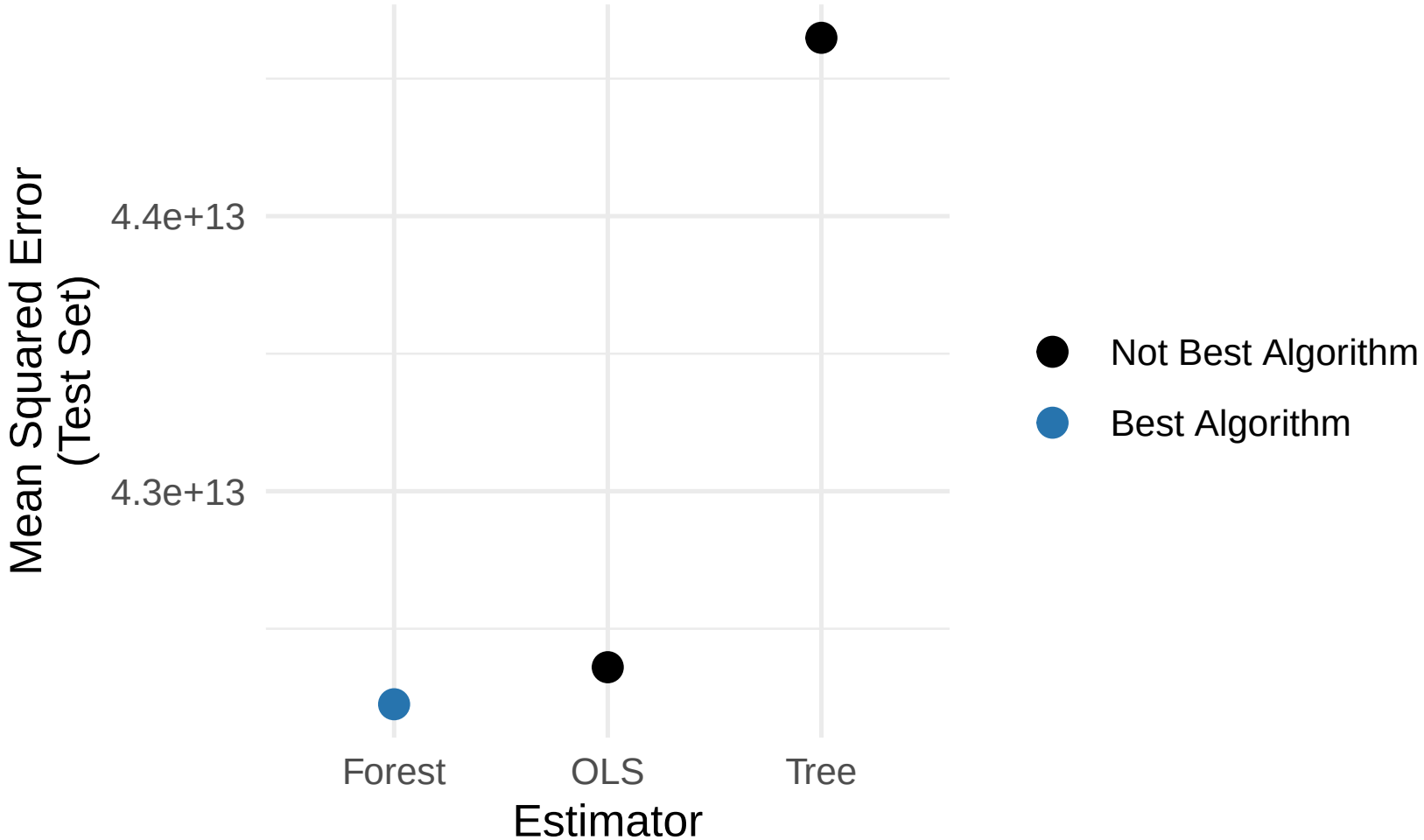


Average prediction error over folds

Out-of-sample predictive performance is not just for tuning parameters.

It can help you choose your algorithm.


Forest
OLS
Tree
Team Average Salary This Season
$10.00m
$7.50m
$5.00m
$2.50m
$2m
$4m
$6m
$8m
Team Average Salary Last Season


Mean Squared Error
(Test Set)
4.4e+13
4.3e+13
Forest
OLS
Tree
Estimator
Not Best Algorithm
Best Algorithm

Writing Task: A possible abstract

Imagine that your results came out really amazing.
Write the abstract of your paper with these imaginary results.

- Minimize jargon. Write for a New York Times reader.
- Emphasize big claims, not buried in statistics

Goal is to ask a high-impact question.
If possible, also write the abstract if results were opposite.

If your abstract is not compelling,
you might consider finding a new research question.

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- Explain the idea of cross-validation

On a different topic, we will close with a writing task.